HKDSE Mathematics 2012 Core Paper1–Suggested Solution

Section A(1)	**[35]**
1. $\dfrac{m^{-12}n^8}{n^3}$ $=\dfrac{n^{8-3}}{m^{12}}$ $=\boldsymbol{\dfrac{n^5}{m^{12}}}$	**[3]**
2. $\dfrac{3a+b}{8}=b-1$ $3a+b=8(b-1)$ $3a+b=8b-8$ $3a=7b-8$ $\boldsymbol{a=\dfrac{7b-8}{3}}$	**[3]**
3. (a) $x^2-6xy+9y^2$ $=\boldsymbol{(x-3y)^2}$	**[1]**
(b) $x^2-6xy+9y^2+7x-21y$ $=(x-3y)^2+7x-21y$ $=(x-3y)^2+7(x-3y)$ $=\boldsymbol{(x-3y)(x-3y+7)}$	**[2]**
4. (a) The daily wage of Ada $=480(1+20\%)$ $=\$\boldsymbol{576}$	**[2]**
(b) Let $\$x$ be the daily wage of Christine. $x(1-20\%)=480$ $x=\dfrac{480}{1-20\%}$ $x=600$ **Thus, Christine has the highest daily wage.**	**[2]**
5. Let x be the number of male guards in the exhibition centre. Then, **the number of female guards in the exhibition centre is** $(\boldsymbol{x+24})$ **.** $\boldsymbol{x+(x+24)=132}$ $2x=108$ $x=54$ Thus, **the number of male guards in the exhibition centre is 54.**	**[4]**

6. **(a)** $\frac{4x+6}{7} > 2(x-3)$

$4x+6 > 14(x-3)$

$10x < 48$

$\boldsymbol{x < \frac{24}{5}}$

$2x - 10 \leq 0$

$\boldsymbol{x \leq 5}$

Thus, the required solution is $x < \frac{24}{5}$ **[3]**

(b) **4** **[1]**

7. **(a)** a

$= 18.1 - 6.8$

$= \mathbf{11.3}$

b

$= 12.1 + 3.2$

$= \mathbf{15.3}$ **[2]**

(b) Note that the longest time taken by the students to finish a 100m race after the training is 15.2 s which is less than the upper quartile of the distribution of the times taken before the training.

Thus, **the claim is agreed.** **[2]**

8. **(a)** $\boldsymbol{\Delta AED \sim \Delta BEC}$

$\frac{AE}{BE} = \frac{DE}{CE}$

$\frac{AE}{8} = \frac{15}{20}$

$\boldsymbol{AE}$ **= 6 cm** **[3]**

(b) $AE^2 + BE^2$

$= 6^2 + 8^2$

$= 10^2$

$= AB^2$

Thus, $\boldsymbol{AC}$ and $\boldsymbol{BD}$ are perpendicular to each other. **[2]**

9. **(a)** Let x cm be the length of AD .

$$\frac{(6+x)(12)}{2}(10)=1020$$

$$x=11$$

Thus, the length of AD is 11cm . **[2]**

(b) CD

$=\sqrt{12^2+(11-6)^2}$

$=13$ cm

The total surface area of the prism $ABCDEFGH$

$=(12+11+13+6)(10)+\frac{(6+11)(12)}{2}(2)$

$=\mathbf{624\ cm^2}$ **[3]**

Section A(2) [35]

10. **(a)** The mean

= 18

The median

= 16 **[2]**

(b) **(i)** The mean

= 18 **[2]**

(ii) Let a and b be the numbers of hours recorded in the two other questionnaires.

Note that $\frac{a+b+19+20}{4}=18$

Therefore, we have $a+b=33$.

If the two medians are the same, then we have $a\leq 16$ and $b\leq 16$.

Hence, we have $a+b\leq 32$.

It is impossible since $a+b=33$

Thus, **it is not possible that the two medians are the same**. **[3]**

11. **(a)** **Let $C=r+sA$, where r and s are· non-zero constants.**

So, we have $r+2s=62$ and $r+6s=74$.

Solving, we have $\mathbf{r=56}$ **and** $\mathbf{s=3}$.

The required cost

$=56+3(13)$

$=\$\mathbf{95}$ **[4]**

(b) Since the volume of the larger can is 8 times that of the can described in (a), the surface area of the larger can is 4 times that of the can described in (a).

The surface area of the larger can

$= (13)(4)$

$= 52 \text{ m}^2$

The required cost

$= 56 + 3(52)$

$= \$\mathbf{212}$ **[2]**

12. (a) The required volume

$= \frac{1}{3}\pi(48)^2(96)$

$= \mathbf{73728\pi \text{ cm}^3}$ **[2]**

(b) (i) The required volume

$= \frac{2}{3}\pi(60)^3$

$= \mathbf{144000\pi \text{ cm}^3}$ **[2]**

(ii) Let h cm be the height of the frustum under the surface of the milk and r cm be the base radius of the circular cone above the surface of the milk.

h

$= \sqrt{60^2 - 48^2}$

$= 36$

$\frac{r}{48} = \frac{96 - 36}{96}$

$r = 30$

The volume of the milk remaining in the vessel

$= 144000\pi - \left(73728\pi - \frac{1}{3}\pi(30)^2(96 - 36)\right)$

$= 88272\pi \text{ cm}^3$

$= 0.2773146667 \text{ m}^3$

$< 0.3 \text{ m}^3$

Thus, the claim is disagreed. **[3]**

13. (a) $k(2)^3 - 21(2)^2 + 24(2) - 4 = 0$

$8k = 40$

$\boldsymbol{k = 5}$ **[2]**

(b) (i) The area of the rectangle $OPQR$

$= \boldsymbol{m(15m^2 - 63m + 72)}$

$= 15m^3 - 63m^2 + 72m$ **[1]**

(ii) Note that the area of the rectangle $OPQR$ is 12.

$15m^3 - 63m^2 + 72m = 12$

$5m^3 - 21m^2 + 24m - 4 = 0$

$\boldsymbol{(m-2)(5m^2 - 11m + 2) = 0}$

$(m-2)^2(5m-1) = 0$

$m = 2$ or $m = \frac{1}{5}$

So, **there are only two different positions of Q such that the area of the rectangle $OPQR$ is 12.**

Thus, there are no three different positions of Q such that the area of the rectangle $OPQR$ is 12 . **[4]**

14. (a) (i) **Γ is parallel to L .** **[1]**

(ii) Note that the **y-intercept of Γ is -2 .**

The slope of L

$= \frac{-1-0}{0-3}$

$= \boldsymbol{\frac{1}{3}}$

The equation of Γ is

$y + 2 = \frac{1}{3}(x - 0)$

$\boldsymbol{x - 3y - 6 = 0}$ **[4]**

(b) (i) Note that the coordinates of **Q are $(6, 0)$** .

Since $6 - 3(0) - 6 = 0$, Γ passes through Q. **[2]**

(ii) Note that both QH and QK are radii of the circle.

Also note that both the heights of ΔAQH and ΔBQK are the distance between L and Γ .

Therefore, the area of ΔAQH is equal to the area of ΔBQK .

Thus, **the required ratio is 1:1 .** **[2]**

Section B	**[35]**

15. (a) The standard deviation

$= 10(1 + 20\%)$

$=$ **12 marks** **[1]**

(b) Let x be the test score and m be the mean of the test scores before the score adjustment.

The standard score before the score adjustment

$= \frac{x - m}{10}$

The standard score after the score adjustment

$= \frac{(x(1 + 20\%) + 5) - (m(1 + 20\%) + 5)}{12}$

$= \frac{1.2(x - m)}{12}$

$= \frac{x - m}{10}$

Thus, there is no change in the standard score of each student due to the score adjustment. **[2]**

16. (a) The required probability

$= \frac{C_4^8 (C_1^2)^4}{C_4^{16}}$

$= \mathbf{\frac{8}{13}}$ **[2]**

(b) The required probability

$= 1 - \frac{8}{13}$

$= \mathbf{\frac{5}{13}}$ **[2]**

17. (a) Note that the radius of C is 10.

Thus, **the equation of C is** $\mathbf{(x - 6)^2 + (y - 10)^2 = 10^2}$ **[2]**

(b) The equation of L is $y = -x + k$.

Putting $y = -x + k$ in $x^2 + y^2 - 12x - 20y + 36 = 0$,

we have $x^2 + (-x + k)^2 - 12x - 20(-x + k) + 36 = 0$

So, we have $2x^2 + (8 - 2k)x + (k^2 - 20k + 36) = 0$.

The x-coordinate of the mid-point of AB

$$=\frac{\frac{-(8-2k)}{2}}{2}$$

$$=\frac{\boldsymbol{k-4}}{\mathbf{2}}$$

The y-coordinate of the mid-point of AB

$$=-\frac{k-4}{2}+k$$

$$=\frac{\boldsymbol{k+4}}{\mathbf{2}}$$

Thus, the required coordinates are $\left(\frac{k-4}{2},\frac{k+4}{2}\right)$. **[5]**

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18. (a) By sine formula, we have

$$\frac{AP}{\sin\angle PBA}=\frac{AB}{\sin\angle APB}$$

$$\frac{AP}{\sin 60^\circ}=\frac{20}{\sin(180^\circ-60^\circ-72^\circ)}$$

$$AP=23.30704256\text{ cm}$$

$$\boldsymbol{AP=23.3\text{ cm}}$$

Thus, the length of AP is 23.3 cm . **[2]**

(b) (i) Let S be the foot of the perpendicular from P to AD.

PS

$=AP\sin\angle PAD$

$=23.30704256\sin 72^\circ$

$=22.1663147\text{ cm}$

AS

$=AP\cos\angle PAD$

$=23.30704256\cos 72^\circ$

$=7.202272239\text{ cm}$

By sine formula, we have

$$\frac{PB}{\sin\angle PAB}=\frac{AB}{\sin\angle APB}$$

$$\frac{PB}{\sin 72^\circ}=\frac{20}{\sin(180^\circ-60^\circ-72^\circ)}$$

$$PB=25.59545552\text{ cm}$$

Let T be the foot of the perpendicular from P to BC.

$PT^2 = PB^2 - AS^2$

$PT^2 = (25.59545552)^2 - (7.202272239)^2$

$PT = 24.56124219$ cm

Note that $\alpha = \angle PTS$

By cosine formula, we have

$$\cos\alpha = \frac{PT^2 + ST^2 - PS^2}{2(PT)(ST)}$$

$$\cos\alpha = \frac{(24\ 56124219)^2 + 20^2 - (22.1663147)^2}{2(24\ 56124219)(20)}$$

$\alpha = 58.59703733°$

$\boldsymbol{\alpha = 58.6°}$

[4]

(ii) Let X be the projection of P on the base $ABCD$.

Then, we have $\beta = \angle PBX$

Note that $PB > PT$.

$\sin\alpha$

$= \frac{PX}{PT}$

$> \frac{PX}{PB}$

$= \sin\angle PBX$

$= \sin\beta$

Since α and β are acute angles, α is greater than β. **[2]**

19. (a) (i) Note that $ab^2 = 254100$ and $ab^4 = 307461$

So, we have $b^2 = \frac{307461}{254100}$

Solving, we have $\boldsymbol{b = 1.1}$ **and** $\boldsymbol{a = 210000}$.

The required weight

$= (210000)(1.1^{(2)(4)})$

$= \mathbf{450153.6501}$ **tonnes** **[4]**

(ii) The total weight of the goods

$= ab^2 + ab^4 + \cdots + ab^{2n}$

$$= \frac{ab^2(b^{2n} - 1)}{b^2 - 1}$$

$= \dfrac{(210000)(1.1)^2((1.1)^{2n}-1)}{1.1^2-1}$

$= \mathbf{1210000((1.1)^{2n}-1)}$ **tonnes**

[2]

(b) **(i)** Note that $A(4) = 450153.65 > 420\,000 = 2a$.

Also note that $(1.1)^{2m} > (1.1)^m$ for any positive integer m .

$A(m+4)$

$= (1.1)^{2m}A(4)$

$> (1.1)^{2m}(2a)$

$> (1.1)^m(2a)$

$= B(m)$

Thus, the claim is agreed. [2]

(ii) Let n be the number of years elapsed since the start of the operation of X.

The total weight of the goods handled by Y

$= 2ab + 2ab^2 + \cdots + 2ab^{n-4}$

$= \left(\frac{2ab(b^{n-4}-1)}{b-1}\right)$ tonnes, where $n > 4$

$$\mathbf{1210000((1.1)^{2n}-1) + \frac{420000(1.1)((1.1)^{n-4}-1)}{1.1-1} > 20000000}$$

$$121(1.1^{2n}) + 462(1.1^{n-4}) - 2583 > 0$$

$$121(1.1^4)(1.1^n)^2 + 462(1.1^n) - 2583(1.1^4) > 0$$

$1.1^n > 3.496831134$ or $1.1^n < -6.104\,70069$ (rejected)

$n \log 1.1 > \log 3.496831134$

$n > 13.13455888$

Note that n is an integer.

Thus, the new facilities should be installed in the 14th year since the start of the operation of X. [5]