HKDSE Mathematics 2022 Core Paper1–Suggested Solution

Section A(1)	**[35]**
1. $\dfrac{(a^3b^{-2})^4}{a^{-5}b^6}$ $=\dfrac{a^{12}b^{-8}}{a^{-5}b^6}$ $=\dfrac{a^{12+5}}{b^{6-(-8)}}$ $=\dfrac{\boldsymbol{a^{17}}}{\boldsymbol{b^{14}}}$	**[3]**
2. $\begin{cases} x+y=456 \\ 7x=y \end{cases}$ $\therefore\ x+7x=456$ Solving, we have $\boldsymbol{x=57}$.	**[3]**
3. $\dfrac{3}{k-9}+\dfrac{2}{5k+6}$ $=\dfrac{3(5k+6)+2(k-9)}{(k-9)(5k+6)}$ $=\dfrac{15k+18+2k-18}{(k-9)(5k+6)}$ $=\dfrac{\boldsymbol{17k}}{\boldsymbol{(k-9)(5k+6)}}$	**[3]**
4. **(a)** $9c^2-6c+1=\boldsymbol{(3c-1)^2}$	**[1]**
(b) $(4a+d)^2-9c^2+6c+1$ $=(4c+d)^2-(9c^2-6c+1)$ $=(4c+d)^2-(3c-1)^2$ $=(4c+d+3c-1)(4c+d-3c+1)$ $=\boldsymbol{(7c+d-1)(c+d+1)}$	**[3]**
5. Let \$$x$ be the cost of the fan. $x\times 26\%=78$ $x=300$ Let \$$y$ be the marked price of the fan. $y\times 70\%=300+78$ $\boldsymbol{y=540}$ $\therefore$ The marked price of the fan is \$540.	**[4]**

6. **(a)** $-2(3x+2) > x+10$

$-6x-4 > x+10$

$-6x-x > 10+4$

$\boldsymbol{x < -2}$

Also, $2x \le 8$

$x \le 4$

$\therefore$ We have $x < -2$ or $x \le -4$

$\therefore$ The solution is $\boldsymbol{x < -2}$ **[3]**

(b) $\mathbf{-3}$ **[1]**

7. **(a)** $\boldsymbol{S'(5, 12)}$.

$\boldsymbol{T'(-3, 7)}$. **[2]**

(b) $m_{S'T'} = \dfrac{12-7}{5-(-3)}$

$= \dfrac{5}{8}$ **[2]**

8. **(a)**

$\angle ACB = \angle CAD$ (alt. $\angle$s, $AD//BC$)

$\angle CAD = \angle ADE$ (alt. $\angle$s, $AC//ED$)

$\angle ACB = \angle ADE$

$\angle ABC = \angle AED$ (given)

$AB = AE$ (given)

$\Delta ABC \cong \Delta AED$ (AAS) **[2]**

(b) $\angle BAC = \angle DAE$

$= 87°$

$\angle ACB = 180° - \angle BAC - \angle ABC$

$= 180° - 87° - 39°$

$= 54°$

$\angle CAD = \angle ACB$

$= 54°$

Note that $AC = AD \Rightarrow ACD = \angle ADC$

$\angle ACD = \dfrac{180° - \angle CAD}{2}$

$= \dfrac{180° - 54°}{2}$

$= \mathbf{63°}$ **[3]**

9. **(a)** **12** **[1]**

(b) Note that $a = 3$ and $b = 5$.

Mean

$= \frac{12(3) + 17(9) + 22(5) + 27(3)}{20}$

=19 minutes **[2]**

(c) The required probability

$= \frac{12}{20}$

$= \frac{\mathbf{3}}{\mathbf{5}}$ **[2]**

Section A(2) [35]

10. **(a)** **Let $f(x) = ax^2 + b$ where a and b are non-zero constant.**

$\therefore \begin{cases} 16a + 4b = 96 \\ 25a - 5b = 15 \end{cases}$

Solving, **we have $a = 3$ and $b = 12$.**

$\therefore\ f(x) = 3x^2 + 12x$. **[3]**

(b) $8(3x^2 + 12x) = 0$
$x = 0$ or -4

$\therefore$ **x-intercept are 0 and -12.** **[1]**

(c) $\because\ 3x^2 + 12x - k = 0$ has two distinct real roots

$\therefore \Delta = 12^2 - 4(3)(-k) > 0$

$144 + 12k > 0$

$\boldsymbol{k > -12}$ **[2]**

11. **(a)** $36 - (20 + a) = 14$

$\boldsymbol{a = 2}$

$30 + b = 31$

$\boldsymbol{b = 1}$ **[3]**

(b) **(i)** The original mode $= 36$

$\because$The frequency of ages apart from age 36 is less than 3,

$\therefore$The new mode = 36

Thus, there is **no change in the mode of the distribution**. **[2]**

(ii) There are two cases.

Case 1: The player of age 17 leaves the football team.

The standard deviation of the distribution $= 7.162537194$

Case 2: The player of age 43 leaves the football team.

The standard deviation of the distribution $= 7.132307207$

$\therefore$ The **greatest possible standard deviation of the distribution is 7.16 .** **[2]**

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12. (a) Coordinates of $G = \left(\frac{154}{2}, \frac{128}{2}\right)$

$= (77, 64)$

$GH = \sqrt{(77-65)^2 + (64-48)^2}$

$= \mathbf{20}$ **[3]**

(b) (i) $GH \perp GP$ **[1]**

(ii) The radius of $C = \sqrt{77^2 + 64^2 - 224}$

$= 99$

The required perimeter

$= 20 + 99 + \sqrt{20^2 + 99^2}$

$= \mathbf{220}$ **[3]**

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13. (a) Volume of the smaller sphere : Volume of the larger sphere = 8: 27 .

The volume of the smaller sphere

$= \frac{4}{3}\pi(9)^3\left(\frac{8}{27}\right)$

= 288π cm^3 **[3]**

(b) $288\pi + \frac{4}{3}\pi(9^3) = \frac{1}{3}\pi(6^2)(10) + V_B$

$V_B = 1140\pi \text{ cm}^3$

$\frac{V_B}{V_A} = \frac{1140\pi}{120\pi} = \frac{19}{2}$

$\left(\frac{r_B}{r_A}\right)^3 = \left(\frac{12}{6}\right)^3 = 8$

$\frac{V_B}{V_A} \neq \left(\frac{r_B}{r_A}\right)^3$

So, A and B are not similar.

Thus, **the claim is not correct**. **[4]**

14. (a) Let $p(x) = (cx + d)(x^2 - 2x + 3) + x + 13$, where c and d are constants.

$p(x) = cx^3 + (d - 2c)x^2 + (3c - 2d + 1)x + (3d + 13)$.

By comparing coefficients, we have $c = 2$ and $3d + 13 = -20$.

Solving, we have $c = 2$ and $d = -11$.

$\therefore p(x) = 2x^3 - 15x^2 + 29x - 20$.

$\therefore$ $\boldsymbol{a = -15}$ **and** $\boldsymbol{b = 29}$. **[3]**

$p(5)$

$= 2(5)^3 - 15(5)^2 + 29(5) - 20$

$= 0$

$\therefore$ $\boldsymbol{x - 5}$ **is a factor of** $\boldsymbol{p(x)}$. **[2]**

(b) $p(x) = 0$

$(x - 5)(2x^2 - 5x + 4) = 0$

$x = 5$ or $2x^2 - 5x + 4 = 0$

$\Delta = (-5)^2 - 4(2)(4)$

$= -7$

< 0

$\therefore 2x^2 - 5x + 4 = 0$ has no irrational roots.

Also note that 5 is not an irrational number.

$\therefore p(x) = 0$ has no irrational roots.

$\therefore$ **The claim is disagreed.** **[3]**

15. (a) The required probability

$= \dfrac{C_2^{10} C_2^{12}}{C_4^{12}}$

$= \boldsymbol{\dfrac{54}{133}}$ **[2]**

(b) The required probability

$= 1 - \dfrac{54}{133}$

$= \boldsymbol{\dfrac{79}{133}}$ **[2]**

16. **(a)** $g(x)$

$= 3x^2 + 12kx + 16k^2 + 8$

$= 3(x^2 + 4kx) + 16k^2 + 8$

$= 3(x^2 + 4kx + 4k^2) + 4k^2 + 8$

$= 3(x + 2k)^2 + 4k^2 + 8$

$\therefore$ **The coordinates of the vertex are** $(-2k, 4k^2 + 8)$. **[2]**

(b) Note that $B(2k, 8k^2 + 16)$.

Also, $AM : MB = 1 : 3$

Coordinates of M

$= \left(\frac{3(-2k) + (2k)}{1+3}, \frac{3(4k^2 + 8) + (8k^2 + 16)}{1+3}\right)$

$= (-k, 5k^2 + 10)$ **[3]**

17. **(a)** Note that $\alpha + \beta = -c$ and $\alpha\beta = -9$

$\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (-c)^2 - 2(-9)$

$= c^2 + 18$ **[3]**

(b) $\alpha^2 + \beta^2 - c^2 = 85 - (\alpha^2 + \beta^2)$

$c^2 + 18 - c^2 = 85 - (c^2 + 18)$

$c^2 = 49$

Note that the 1st term = 49 and the common difference = 18

$\frac{n}{2}\left(2(49) + 18(n-1)\right) > 2 \times 10^6$

$9n^2 + 40n - 2 \times 10^6 > 0$

$n < \frac{-40 - \sqrt{40^2 - 4(9)(-2\times 10^6)}}{2(9)}$ or $n > \frac{-40 + \sqrt{40^2 - 4(9)(-2\times 10^6)}}{2(9)}$

$n < -473.6319808$ or $n > 469.1875364$

$\therefore$ **The least value of n is 470 .** **[4]**

18. **(a)** **(i)** $QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos\angle QPR$

$QR^2 = 30^2 + 25^2 - 2(30)(25)\cos 95°$

$QR = 40.69070673$

$QR = 40.7$ **cm** **[2]**

(ii) $\dfrac{\sin\angle PQR}{PR} = \dfrac{\sin\angle QPR}{QR}$

$\dfrac{\sin\angle PQR}{25} = \dfrac{\sin 95°}{40.69070673}$

$\angle PQR = 37.73809375^\circ$ or $\angle PQR = 142.261906°$ (rejected)

$\therefore \boldsymbol{\angle PQR = 37.7^\circ}$ **[2]**

(b) $PM^2 = PQ^2 + QM^2 - 2(PQ)(QM)\cos\angle PQR$

$PM^2 = 30^2 + \left(\dfrac{40.69070673}{2}\right)^2 - 2(30)\left(\dfrac{40.69070673}{2}\right)\cos 37.73809375°$

$PM = 18.66993831$ cm

Let S and N be the projections of R and M on the horizontal ground respectively.

$MN = \dfrac{1}{2}RS$

$= \dfrac{1}{2}PR\sin 70°$

$= \dfrac{1}{2}(25)\sin 70°$

$= 11.74615776$ cm

Note that the angle between PM and the horizontal ground is $\angle MPN$.

$\sin\angle MPN = \dfrac{MN}{PM}$

$\sin\angle MPN = \dfrac{11.74615776}{18.66993831}$

$\angle MPN = 38.98730493°$

$< 40°$

$\therefore$ **The claim is not correct.** **[3]**

19. (a) m_{AG}

$= \dfrac{112 - 12}{83 - 158}$

$= -\dfrac{4}{3}$

The required equation is

$y - 12 = -\dfrac{4}{3}(x - 158)$

$\boldsymbol{4x + 3y - 668 = 0}$ **[2]**

(b) Radius of C is 75 .

$\because$ 83 + 75 = 158

$\therefore$ AP is vertical or AQ is vertical.

$\therefore$ $P(158,112)$ or $Q(158,112)$

$\because$ $\Delta AGP \cong \Delta AGQ$ and $AG \perp PQ$.

$\therefore$ $m_{PQ} = -\frac{1}{m_{AG}} = \frac{3}{4}$.

The equation of PQ is $y - 112 = \frac{3}{4}(x - 158)$.

$$\begin{cases} y - 112 = \dfrac{3}{4}(x - 158) \\ 4x + 3y - 668 = 0 \end{cases}$$

Solving, we have $x = 110$ and $y = 76$.

Thus, the coordinates of the point of intersection of AG and PQ are **(110, 76)** **[3]**

(c) Let B and r be the centre and the radius of the inscribed circle of ΔAPQ respectively.

Note that $AP = AQ$ and B lies on AG .

The x-coordinate of B

$= 158 - r$

The y-coordinate of B

$= -\frac{4}{3}(158 - r) + \frac{668}{3}$

$= \frac{4r}{3} + 12$

$\therefore B\left(158 - r, \frac{4r}{3} + 12\right)$

The distance between and the point of intersection of AG and PQ is r.

$$((158 - r) - 110)^2 + \left(\left(\frac{4}{3} + 12\right) - 76r\right)^2 = r^2$$

$$\frac{16}{9}r^2 - \frac{800}{3}r + 6400 = 0$$

$$r^2 - 150r + 3600 = 0$$

$r = 30$ or $r = 120$ (rejected)

$\therefore$ B(128,52) .

$\therefore$ The required equation is $(\boldsymbol{x} - \mathbf{128})^2 + (\boldsymbol{y} - \mathbf{52})^2 = \mathbf{30}^2$. **[4]**

(d) Note that $\angle APG = \angle AQG = 90°$ and $\angle APG + \angle AQG = 180°$.

$\therefore$ *APGQ* is a cyclic quadrilateral

and *AG* is a diameter of the circumcircle of ΔAPQ .

The radius of the circumcircle of ΔAPQ

$= \frac{1}{2}\sqrt{(83-158)^2+(112-12)^2}$

$= \frac{125}{2}$

By (c), the radius of the inscribed circle of ΔAPQ is 30.

Area of the inscribed circle : Area of the circumcircle of ΔAPQ

$= 30^2 : \left(\frac{125}{2}\right)^2$

$= 144 : 625$

$\neq 1 : 4$

$\therefore$ **The claim is disagreed.** **[3]**