HKDSE Mathematics 2021 Core Paper 1 – Suggested Solution

Section A(1)	[35]
1. $(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$ $= (\alpha\beta^3)(\alpha^{-10}\beta^{20})$ $= \alpha^{-9}\beta^{23}$ $= \boldsymbol{\frac{\beta^{23}}{\alpha^9}}$	[3]
2. $\frac{4-3a}{b} = 5$ $4 - 3a = 5b$ $-3a = 5b - 4$ $\boldsymbol{a = \frac{4-5b}{3}}$	[3]
3. (a) $6x^2 + xy - 2y^2$ $= (\boldsymbol{2x - y})(\boldsymbol{3x + 2y})$	[1]
(b) $8x - 4y - 6x^2 - xy + 2y^2$ $= 8x - 4y - (2x - y)(3x + 2y)$ $= 4(2x - y) - (2x - y)(3x + 2y)$ $= (\boldsymbol{2x - y})(\boldsymbol{4 - 3x - 2y})$	[2]
4. (a) $\frac{7(x-2)}{5} + 11 > 3(x-1)$ $7x - 14 > 15x - 70$ $-8x > -56$ $\boldsymbol{x < 7}$ Also, $x + 4 \geq 0$ $x \geq -4$ $\therefore$ The required range is $\boldsymbol{-4 \leq x < 7}$.	[3]
(b) **6**	[1]
5. **Let x and $3x$ and $3x$ be the number of stickers owned by the girl and the boy respectively and boy respectively.$2(3x - 20) = x + 20$** $\boldsymbol{2(3x - 20) = x + 20}$ $6x - 40 = x + 20$ $x = 12$ $\therefore$ **The required number= 12+36 = 48 .**	[4]

6. Let $\$x$ be the cost of the shirt.

$(x + 80) \times 90\% = x \times (1 + 30\%)$

$0.9x + 72 = 1.3x$

$x = 180$

$\therefore$ **The required marked price is $260.** **[4]**

7. (a) $\angle POQ$

$= 140° - 80°$

$= \mathbf{60°}$ **[1]**

(b) $\because$ Note that ΔOPQ is an equilateral triangle

$\therefore \boldsymbol{r = 21}$ **[1]**

(c) The perimeter of ΔOPQ

$= 3(21)$

$= \mathbf{63}$ **[2]**

8. (a)

$\angle CAE = \angle BDE$ (given)

$\angle AEC = \angle DEB$ (common $\angle$)

$\angle ACE = \angle DBE$ ($\angle$ sum of Δ)

$\Delta ACE \sim \Delta DBE$ (AAA) **[2]**

(b) (i) $\because AC^2 + AE^2$

$= 25^2 + 60^2$

$= 4225$

$= 65^2$

$= CE^2$

$\therefore$ $\boldsymbol{\Delta ACE}$ **is a right-angled triangle.** **[1]**

(ii) $\frac{DE}{AE} = \frac{BD}{AC}$

$\frac{DE}{60} = \frac{15}{25}$

$DE = 36$ cm

Note that $\angle BDE = 90°$

The area of ΔBDE

$= \frac{15(36)}{2}$

$= \mathbf{270}$ **cm²** **[2]**

9. **(a)** $\dfrac{12+k+16}{12+k+16+9+11+4}=\dfrac{7}{10}$

$\boldsymbol{k=28}$ **[2]**

(b) The range

$=\mathbf{5}$

The inter-quartile range

$=\mathbf{2}$

The standard deviation

$=\mathbf{1.43}$ **[3]**

Section A(2) [35]

10. **(a)** Let $f(x)=k_1(x+4)^2+k_2$

$\because f(-3)=0$ and $f(2)=105$

$\therefore \begin{cases} m+n=0 \\ 36m+n=105 \end{cases}$

Solving, we have $k_1=3$ and $k_2=-3$.

$\therefore f(x)=3(x+4)^2-3$

$\therefore \boldsymbol{f(0)=45}$ **[3]**

(b) **(i)** 48 **[1]**

(ii) Put $y=0$

$3(x+4)^2=0$

$\boldsymbol{x=-4}$

$\therefore$The x-intercept of G is -4 . **[2]**

11. **(a)** The mean

$=\dfrac{1(15)+2(9)+3(2)+4(5)+5(4)+6(2)+7(5)}{15+9+2+5+4+2+5}$

$=\mathbf{3}$ **[2]**

(b) The median and the mode are 2 and 1 respectively.

$\therefore$**The median and the mode of the distribution are not equal**. **[2]**

(c) **(i)** **42** **[1]**

(ii) **11** **[1]**

(iii) **10** **[1]**

12. (a) Let $p(x) \equiv (x^2+x+1)(2x^2-37)+cx+c-1$.

$$p(5)=0$$
$$(5^2+5+1)(2(5^2)-37)+5c+c-1=0$$
$$6c+402=0$$
$$\boldsymbol{c=-67}$$ **[3]**

(b)
$p(x)$
$=(x^2+x+1)(2x^2-37)-67x-68$
$=2x^4+2x^3-35x^2-104x-105$

$p(-3)$
$=2(-3)^4+2(-3)^3-35(-3)^2-104(-3)-105$
$=0$

$\therefore$ **$x+3$ is a factor of $p(x)$.** **[1]**

(c) By (b), we have $p(x)=2x^4+2x^3-35x^2-104x-105$.

By long division, we have

$p(x)=(x+3)(x-5)(2x^2+6x+7)$.

$$p(x)=0$$
$$(x+3)(x-5)(2x^2+6x+7)=0$$
$$x=-3,\ x=5 \quad \text{or} \quad 2x^2+6x+7=0$$

$\Delta=6^2-4(2)(7)$
$=-20$
<0

$\therefore$ The roots of the equation $\boldsymbol{2x^2+6x+7=0}$ are not real numbers.

$\therefore$ The claim is not correct. **[3]**

13. (a) Note that $G(6, 8)$.

OG
$=\sqrt{(6-0)^2+(8-0)^2}$
$=\mathbf{10}$ **[2]**

(b) The radius of C
$=\frac{1}{2}\sqrt{(-12)^2+(-16)^2+4(69)}$
$=13$
$>OG$

$\therefore$ **O lies inside C.** **[1]**

(c) ∵ Both M and N lie on Γ

∴ $OM = GM$ and $ON = GN$

Also note that $GM = GN = 13$.

Therefore, we have $OM = MG = GN = NO$

So, the quadrilateral $OMGN$ is a rhombus.

Let A be the mid-point of OG.

Note that $OG \perp MA$

$$\begin{aligned} & GA \\ &= \frac{1}{2}OG \\ &= \frac{1}{2}(10) \\ &= 5 \end{aligned}$$

$$\begin{aligned} & MA \\ &= \sqrt{GM^2 - GA^2} \\ &= \sqrt{13^2 - 5^2} \\ &= 12 \end{aligned}$$

The required area

$$\begin{aligned} &= 4\left(\frac{1}{2}(GA)(MA)\right) \\ &= 4\left(\frac{1}{2}(5)(12)\right) \\ &= \mathbf{120} \end{aligned}$$

[4]

14. (a) Let r cm be the base radius of Y.

$$\frac{24\pi r^2}{3} = 800\pi$$

$$r = 10$$

∴ the base radius of Y is **10 cm** .

[2]

(b) V_z

$= \pi(10^2)(20) + 800\pi$

$= 2800\pi \text{ cm}^3$

$\left(\frac{r_y}{r_z}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\frac{V_y}{V_z} = \frac{800}{2800} = \frac{2}{7}$

$\frac{V_y}{V_z} \neq \left(\frac{r_y}{r_z}\right)^3$

$\therefore$ ***Y* and *Z* are not similar.** **[3]**

(c) The curved surface area of X

$= 2\pi(10)(20)$

$= 400\pi \text{ cm}^2$

The curved surface area of Y

$= \pi(10)\sqrt{10^2 + 24^2}$

$= 260\pi \text{ cm}^2$

Let h cm be the height of Z.

$\frac{\pi(20^2)(h)}{3} = 2800\pi$

$h = 21$

$\therefore$ the height of Z is 21 cm .

The curved surface area of Z

$= \pi(20)(\sqrt{20^2 + 21^2})$

$= 580\pi \text{ cm}^2$

The sum of the curved surface area of X and the curved surface area of Y

$= 400\pi + 260\pi$

$= 660\pi \text{ cm}^2$

$> 580\pi \text{ cm}^2$

$\therefore$ **the claim is agreed.** **[3]**

Section B [35]

15. (a) The required number

$= 10!\,10!$

$= \mathbf{3\ 628\ 800}$ **[1]**

(b) The required probability

$= \frac{7!\,C_3^8 \times 3!}{10!}$

$= \mathbf{\frac{7}{15}}$

[3]

16. (a) m_{L_1}

$= \frac{6-3}{2-0}$

$= \frac{3}{2}$

The equation of L_1 is

$$y - 3 = \frac{3}{2}(x - 0)$$

$\mathbf{3x - 2y + 6 = 0}$

The equation of L_2 is

$$y - 6 = -\frac{2}{3}(x - 2)$$

$2x + 3y - 22 = 0$

$\therefore$ the system of inequalities is $\begin{cases} \mathbf{3x - 2y + 6 \geq 0} \\ \mathbf{2x + 3y - 22 \leq 0} \\ \mathbf{y \geq 0} \end{cases}$.

[3]

(b) Note that the vertices of R are the points $(-2, 0)$, $(2, 6)$ and $(11, 0)$.

For (-2,0), we have $8x - 5y = -16$.

For (2,6), we have $8x - 5y = -14$.

For (11,0) , we have $8x - 5y = 88$.

$\therefore$ **The least value of $8x - 5y$ is -16**

[2]

17. (a) Let $A(n) = a + (n-1)d$, where a and d are constant.

Note that $a + 4d = 26$ and $a + 11d = 61$

Solving, we have $a = 6$ and $d = 5$.

Thus, $A(1) = a = 6$.

[2]

(b)

$$\log_8\big(G(1)G(2)G(3)\cdots G(k)\big) < 999$$

$$\frac{\log_2\big(G(1)G(2)G(3)\cdots G(k)\big)}{\log_2 8} < 999$$

$$\log_2\big(G(1)G(2)G(3)\cdots G(k)\big) < 2997$$

$$\log_2 G(1) + \log_2 G(2) + \log_2 G(3) + \cdots + \log_2 G(k) < 2997$$

$$A(1) + A(2) + A(3) + \cdots + A(k) < 2997$$

$$\frac{k}{2}\big(2(6) + (k-1)(5)\big) < 2997$$

$$5k^2 + 7k - 5994 < 0$$

$$-35.33076667 < k < 33.93076667$$

$\therefore$ **The greatest value of k is 33 .** **[5]**

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18. (a) Let P be a point lying on AD such that $AB//PC$.

By sine formula, we have

$$\frac{CD}{\sin\angle CPD} = \frac{CP}{\sin\angle CDP}$$

$$\frac{CD}{\sin 50°} = \frac{45}{\sin 70°}$$

$$CD = 36.68433611$$

$$\boldsymbol{CD = 36.7}\ \textbf{cm}$$ **[2]**

(b) (i) $AE = AB\cos\angle BAE = 45\cos 50° = 28.92544244$ cm

$DE = BC + CD\cos\angle CDE$

$= 40 + 36.68433611\cos 70°$

$= 52.54678189$ cm

$AD = \sqrt{AE^2 + DE^2}$

$= 59.98204321$ cm

Note that $\angle ABC = 90°$.

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{45^2 + 40^2}$

$= 60.20797289\ cm$

By cosine formula, we have

$$\cos\angle CAD = \frac{AC^2 + AD^2 - CD^2}{2(AC)(AD)}$$

$\angle CAD = 35.54210789°$

$\boldsymbol{\angle CAD = 35.5°}$ **[3]**

(ii) Let N be the foot of the perpendicular from A to CD .

The angle between the plane ACD and the plane $BCDE$ is $\angle ANE$.

$$\frac{(AN)(CD)}{2}=\frac{(AC)(AD)\sin\angle CAD}{2}$$

$$\frac{(AN)(36.68433611)}{2}=\frac{(60.20797289)(59.98204321)\sin 35.54210789^\circ}{2}$$

$$AN=57.22631076\text{ cm}$$

$$\sin\angle ANE=\frac{AE}{AN}$$

$$\sin\angle ANE=\frac{28.92544244}{57.22631076}$$

$$\angle ANE=30.36169732^\circ$$

$$>30^\circ$$

$\therefore$ the angle between the plane ACD and the plane $BCDE$ exceeds 30° . **[2]**

19. (a) $f(x)$

$=x^2-12kx-14x+36k^2+89k+53$

$=x^2-2(6k+7)x+(6k+7)^2-(6k+7)^2+36k^2+89k+53$

$=(x-6k-7)^2+5k+4$

$\therefore$ $\boldsymbol{Q(6k+7,5k+4)}$ **.** **[2]**

(b) $\boldsymbol{(7-6k,5k+4)}$ **[1]**

(c) (i)

$$m_{QS}=\frac{5k+4-(4-3k)}{6k+7-7}=\frac{4}{3}$$

The required equation is

$$y-(4-3k)=\frac{4}{3}(x-7)$$

$$\boldsymbol{4x-3y-9k-16=0}$$

(ii) Let r be the radius of C .

Note that $QS=RS$.

$\therefore$ The coordinates of the centre of C are $(7,5k+4-r)$.

$\therefore$ The equation of C is $(x-7)^2+(y-5k-4+r)^2=r^2$

Putting $y=\frac{4x-16}{3}-3k$ into $(x-7)^2+(y-5k-4+r)^2=r^2$

$$(x-7)^2+\left(\frac{4x-16}{3}-3k-5k-4+r\right)^2=r^2$$

$$25x^2+(24r-192k-350)x+576k^2-144kr+1344k-168r+1225=0$$

$\because QS$ is a tangent to C

$\therefore \Delta = (24r - 192k - 350)^2 - 4(25)(576k^2 - 144kr + 1344k - 168r + 1225) = 0$

$r^2 + 9kr - 36k^2 = 0.$

$r = 3k$ or $r = -12k$ (rejected).

$\therefore$ the equation of C is

$(x - 7)^2 + (y - 5k - 4 + 3k)^2 = (3k)^2$

$\boldsymbol{(x - 7)^2 + (y - 2k - 4)^2 = 9k^2}$ **[4]**

(iii) For ST // VU, $m_{UV} = m_{QS}$.

$$\frac{-14 - (2k + 4)}{-29 - 7} = \frac{4}{3}$$

$\boldsymbol{k = 15}$.

$\therefore S(7, -41)$ and $U(7, 34)$

$$m_{SV} = \frac{-14 - (-41)}{-29 - 7} = -\frac{3}{4}$$

Note that $m_{QS} \times m_{SV} = -1$

$\therefore ST \perp SV$

$\because ST \perp TU$

$\therefore SV$ // TU.

$\because$ When $k = 15$, we have ST // VU, SV // TU and $ST \perp TU$.

$\therefore$ **it is possible that *STUV* is a rectangle.** **[3]**