HKDSE Mathematics 2020 Core Paper 1 – Suggested Solution

Section A(1)	**[35]**

1. $\dfrac{(mn^{-2})^5}{m^{-4}}$

$= \dfrac{m^5 n^{-10}}{m^{-4}}$

$= \dfrac{m^{5-(-4)}}{n^{10}}$

$= \boldsymbol{\dfrac{m^9}{n^{10}}}$ **[3]**

2. **(a)** $\alpha^2 + \alpha - 6$

$= \boldsymbol{(\alpha + 3)(\alpha - 2)}$ **[1]**

(b) $\alpha^4 + \alpha^3 - 6\alpha^2$

$= \alpha^2(\alpha^2 + \alpha - 6)$

$= \boldsymbol{\alpha^2(\alpha + 3)(\alpha - 2)}$ **[2]**

3. **(a)** **600** **[1]**

(b) **534.76** **[1]**

(c) **530** **[1]**

4. $a : b$

$= 6 : 7$

$= 12 : 14$

$a : c$

$= 4 : 3$

$= 12 : 9$

$a : b : c = 12 : 14 : 9$

Let $a = 12k$, $b = 14k$ and $c = 9k$, where k is a non-zero constant.

$\dfrac{b + 2c}{a + 2b}$

$= \dfrac{14k + 2(9k)}{12k + 2(14k)}$

$= \boldsymbol{\dfrac{4}{5}}$ **[3]**

5. **Let x and $(1+28\%)x$ be the number of female and male applicants respectively.**

$(1+28\%)x - x = 91$

$0.28x = 91$

$x = 325$

$\therefore$ The number of male applicants is **416** . **[4]**

6. **(a)**

$3 - x > \frac{7-x}{2}$

$6 - 2x > 7 - x$

$-2x + x > 7 - 6$

$x < -1$

Also,

$5 + x > 4$

$x > -1$

$\therefore x \neq -1$ **[3]**

(b) -2 **[1]**

7. **(a)** $\because 4x^2 + 12x + c = 0$ has equal roots

$\therefore \Delta = 12^2 - 4(4)c = 0$

$c = 9$ **[3]**

(b) $p(x) - 169 = 0$

$4x^2 + 12x - 160 = 0$

$4(x+8)(x-5) = 0$

$x = -8$ or $x = 5$

$\therefore x$-intercepts are -8 **and** 5 . **[2]**

8. **(a)**

$\angle AEC$

$= \angle ADB$

$= 42°$

$\angle AEB$

$= \angle CAE$

$= 30°$

$\angle BEC$

$= \angle AEC - \angle AEB$

$= 42° - 30°$

$= 12°$ **[3]**

(b) $\angle DCE$

$= \angle BDC$

$= \theta$

$\angle CFE$

$= 180° - \angle BEC - \angle DCE$

$= 180° - 12° - \theta$

$= \mathbf{168° - \theta}$ **[2]**

9. (a) The mean = **5.4**

The median = **5.5**

The standard deviation = **0.917** **[3]**

(b) The new median = 5

The decrease in the median = 5.5 – 5 = **0.5** **[2]**

Section A(2) [35]

10. (a) Let $\boldsymbol{P = a + bh^3}$

$\therefore \begin{cases} a + 27b = 59 \\ a + 343b = 691 \end{cases}$

Solving, we have $\boldsymbol{a = 5}$ **and** $\boldsymbol{b = 2}$ **.**

The required price

$= 5 + 2(4^3)$

$= \$\mathbf{133}$ **[4]**

(b) When $h = 5$,

$P = 5 + 2(5^3)$

$= 255$

$> 2(133)$

$\therefore$The claim **is not correct.** **[2]**

11. (a) The range $= 50 + w - 11$

$= (w + 39)$ grams

The inter-quartile range $= 38 - 23$

$= 15$ grams

$w + 39 = 3(15)$

$\boldsymbol{w = 6}$ **[4]**

(b) The mode of the distribution is 38 grams.

The required probability

$= \frac{6}{20}$

$= \mathbf{\frac{3}{10}}$

[2]

12. **(a)** The volume of the middle part of the circular cone

$= \frac{1}{3}\pi(15^2)(36)\left(\frac{2^3-1^3}{3^3}\right)$

$= \mathbf{700\pi}$ **cm³**

[3]

(b) The curved surface area of the middle part of the circular cone

$= \pi(15)\left(\sqrt{15^2+36^2}\right)\left(\frac{2^2-1^2}{3^2}\right)$

$= \mathbf{195\pi}$ **cm²**

[3]

13. **(a)** Let $f(x) = (x^2-1)q(x) + (kx+8)$, where $q(x)$ is a polynomial.

$f(1) = 0$

$(1^2-1)q(1) + (k+8) = 0$

$\boldsymbol{k = -8}$

[3]

(b) Note that $f(x) = (x^2-1)q(x) + (-8x+8)$.

$\therefore f(-1) = ((-1)^2-1)q(-1) + \left((-8)(-1)+8\right) = 16$.

Now let $f(x) = (x-1)(x+3)(ax+b)$, where a and b are constants.

$f(0) = 24$

$(-1)(3)(b) = 24$

$\boldsymbol{b = -8}$

$f(-1) = 16$

$(-1-1)(-1+3)(-a-8) = 16$.

$\boldsymbol{a = -4}$ **.**

$\therefore f(x) = (x-1)(x+3)(-4x-8)$.

$\therefore$ The roots of $f(x) = 0$ are $1, -3$ and -2 .

$\because$ All the roots of the equation $f(x) = 0$ are integers.

$\therefore$ **The claim is correct.**

[5]

14. **(a)** The x-coordinate of G

$= \frac{-10+30}{2}$

$= 10$

The radius of C

$= \sqrt{(-10-10)^2 + (0+15)^2}$

$= 25$

$\therefore$ The equation of C is $\boldsymbol{(x-10)^2 + (y+15)^2 = 25^2}$ **[3]**

(b) **(i)** Γ // L **[1]**

(ii) m_Γ

$= m_L$

$= \frac{0+15}{30-10}$

$= \frac{3}{4}$

The equation of Γ is

$y - 0 = \frac{3}{4}(x - (-10))$

$\boldsymbol{3x - 4y + 30 = 0}$ **[2]**

(iii) $\tan \angle ABG = \frac{3}{4}$

$\angle ABG = 36.86989765°$

Note that $\angle BAH = \angle ABG$ and $\angle BAG = \angle ABG$.

$\angle GAH$

$= \angle BAH + \angle BAG$

$= 2\angle ABG$

$> 2(35°)$

$> 70°$

$\therefore$ The claim is **disagreed** **[3]**

Section B **[35]**

15. **(a)** The required probability

$= \frac{C_4^7 + C_4^9}{C_4^{19}}$

$= \boldsymbol{\frac{161}{3876}}$ **[3]**

(b) The required probability

$= 1 - \frac{161}{3876}$

$= \mathbf{\frac{3715}{3876}}$

[2]

16. (a) Let a and r be the first term and the common ratio of the geometric sequence respectively.

$\begin{cases} ar^2 = 144 \\ ar^5 = 486 \end{cases}$

Solving, we have $r = 1.5$

$\therefore a = 64$

$\therefore$ The first term of the sequence is **64**

[2]

(b) $64 + 64(1.5) + 64(1.5^2) + \cdots + 64(1.5^{n-1}) > 8 \times 10^{18}$

$$\frac{64(1.5^n - 1)}{1.5 - 1} > 8 \times 10^{18}$$

$$1.5^n > 6.25 \times 10^{16} + 1$$

$$\log 1.5^n > \log(6.25 \times 10^{16} + 1)$$

$$n \log 1.5 > \log(6.25 \times 10^{16} + 1))$$

$$n > 95.38167941$$

$\therefore$The least value of n is **96 .**

[3]

17. (a) $g(x)$

$= x^2 - 2kx + 2k^2 + 4$

$= x^2 - 2kx + k^2 + k^2 + 4$

$= (x - k)^2 + k^2 + 4$

$\therefore$ The coordinates of the vertex are $(\boldsymbol{k}, \boldsymbol{k^2} + \mathbf{4})$

[2]

(b) Note that $\boldsymbol{D}(\boldsymbol{k} - \mathbf{2}, \boldsymbol{k^2} + \mathbf{4})$ **and** $\boldsymbol{E}(\boldsymbol{k} + \mathbf{2}, -\boldsymbol{k^2} - \mathbf{4})$.

Denote the point $(0, 3)$ by C .

CD^2

$= \left((k - 2) - 0\right)^2 + \left((k^2 + 4) - 3\right)^2$

$= k^4 + 3k^2 - 4k + 5$

CE^2

$= (k + 2 - 0)^2 + \left((-k^2 - 4) - 3\right)^2$

$= k^4 + 15k^2 + 4k + 53$

$$CD^2 = CE^2$$

$$k^4 + 3k^2 - 4k + 5 = k^4 + 15k^2 + 4k + 53$$

$$3k^2 + 2k + 12 = 0$$

$\Delta = 2^2 - 4(3)(12) = -140 < 0.$

$\therefore 3k^2 + 2k + 12 = 0$ has no real roots.

$\therefore$ **There is no such a point F .** **[4]**

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18. (a) $\angle TUV = \angle TWU$ ($\angle$ in alt. segment)

$\angle UTV = \angle UTW$ (common angle)

$\angle TVU = \angle TUW$ ($\angle$ sum of Δ)

$\Delta UTV \sim \Delta WTU$ (AAA) **[2]**

(b) (i)

$$\frac{TW}{TU} = \frac{TU}{TV}$$

$$\frac{TV + VW}{TU} = \frac{TU}{TV}$$

$$\frac{325 + VW}{780} = 325$$

$$VW = 1547 \text{ cm}$$

$\therefore$ **The circumference of C is 1547π cm .** **[2]**

(ii) By (a), we have $UV: UW = TV: TU = 325: 780 = 5: 12$.

$\because$ VW is a diameter of C

$\therefore \angle VUW = 90°$.

By Pyth. Theorem, $UV: UW: VW = 5: 12: 13.$

$$UV = (1547)\left(\frac{5}{13}\right)$$
$$= 595 \text{ cm}$$

$$UW = (1547)\left(\frac{12}{13}\right)$$
$$= 1428 \text{ cm}$$

The perimeter of ΔUVW

$= 595 + 1428 + 1547$

$= 3570$ cm

$= 35.7$ m

> 35 m

$\therefore$ **The claim is agreed.** **[3]**

19. **(a)** By sine formula, we have

$$\frac{PR}{\sin \angle PQR} = \frac{PQ}{\sin \angle PRQ}$$

$$\frac{PR}{\sin 30°} = \frac{60}{\sin 55°}$$

$$PR = 36.62323766 \text{ cm}$$

$\because \angle QPR = 180 - 55° - 30° = 95°$

$\therefore \angle RPS = 120° - 95° = 25°$

By cosine formula, we have

$$RS^2 = PS^2 + PR^2 - 2(PS)(PR)\cos \angle RPS$$

$$RS^2 = 40^2 + 36.62323766^2 - 2(40)(36.62323766)\cos 25°$$

$$RS = 16.90879944$$

$$\boldsymbol{RS = 16.9 \text{ cm}}$$

$\therefore$ The length of RS is 16.9 cm . **[3]**

(b) The area of the paper card

$$= \frac{1}{2}(PQ)(PR)\sin \angle QPR + \frac{1}{2}(PR)(PS)\sin \angle RPS$$

$$= \frac{1}{2}(60)(36.62323766)\sin 95° + \frac{1}{2}(36.62323766)(40)\sin 25°$$

$$= 1404.069236$$

$$= \mathbf{1400 \text{ cm}^2}$$ **[2]**

(c) **(i)** Let H be the foot of the perpendicular from P to QR.

$$PH = PQ \sin \angle PQH$$

$$= 60 \sin 30°$$

$$= 30 \text{ cm}$$

Denote the projection of P on the horizontal ground by G .

So, the angle between the paper card and the horizontal ground is $\angle PHG$.

$\therefore \angle PHG = 32°$.

$$PG = PH \sin \angle PHG$$

$$= 30 \sin 32°$$

$$= 15.89757793$$

$$= \mathbf{15.9 \text{cm}}$$

$\therefore$ The shortest distance from P to the horizontal ground is 15.9 cm . **[3]**

(ii) Denote the projection of S on the horizontal ground by K .

Let T be the point at which PS produced and QR produced meet.

Note that $\Delta SKT \sim \Delta PGT$ and $PT = PQ$.

$\therefore SK = \left(\frac{PT-PS}{PT}\right)PG = \left(\frac{PQ-PS}{PQ}\right)PG = \left(\frac{60-40}{60}\right)PG = \frac{1}{3}PG$.

By (c)(i), we have

$SK = 10\sin 32°$ cm .

Note that the angle between RS and the horizontal ground is $\angle SRK$.

$\sin\angle SRK = \frac{SK}{RS}$

$$\sin\angle SRK = \frac{10\sin 32°}{16.90879944}$$

$$\angle SRK = 18.26416068°$$

$$\leq 20°$$

$\therefore$ **The claim is correct.** **[4]**