HKDSE Mathematics 2019 Core Paper 1 – Suggested Solution

Section A(1)		**[35]**
1.	$9(h+6k)=7h+8$ $9h+54k=7h+8$ $9h-7h=8-54k$ $\boldsymbol{h=4-27k}$	**[3]**
2.	$\dfrac{3}{7x-6}-\dfrac{2}{5x-4}$ $=\dfrac{3(5x-4)-2(7x-6)}{(7x-6)(5x-4)}$ $=\dfrac{15x-12-14x+12}{(7x-6)(5x-4)}$ $=\boldsymbol{\dfrac{x}{(7x-6)(5x-4)}}$	**[3]**
3.	$24^2+(13+r)^2=(17-3r)^2$ $576+169+26r+r^2=289-102r+9r^2$ $8r^2-128r-456=0$ $8(r+3)(r-19)=0$ $\boldsymbol{r=-3}$ **or** $\boldsymbol{r=19}$ **(rejected)** $\therefore r=-3$	**[3]**
4.	**(a)** $4m^2-9$ $=\boldsymbol{(2m+3)(2m-3)}$	**[1]**
	(b) $2m^2n+7mn-15n$ $=n(2m^2+7m-15)$ $=\boldsymbol{n(2m-3)(m+5)}$	**[1]**
	(c) $4m^2-9-2m^2n-7mn+15n$ $=(2m+3)(2m-3)-n(2m-3)(m+5)$ $=\boldsymbol{(2m-3)(2m-mn-5n+3)}$	**[2]**
5.	**(a)** Let \$$m$ be the marked price of the wallet. $(1-25\%)m=690$ $\boldsymbol{m=920}$ $\therefore$ The required price is \$920.	**[2]**
	(b) Let \$$c$ be the cost of the wallet. $(1+15\%)c=690$ $\boldsymbol{c=600}$ $\therefore$ The required cost is \$600.	**[2]**

6. **(a)** $\frac{7x+26}{4} \le 2(3x-1)$

$7x+26 \le 24x-8$

$-17x \le -34$

$\boldsymbol{x \ge 2}$ **[2]**

(b) $45-5x \ge 0$

$\boldsymbol{x \le 9}$

$\therefore 2 \le x \le 9$

$\therefore$ **The required number is 8.** **[2]**

7. **Let $13k$ and $6k$ be the original number of adults and children in the playground respectively.**

$\boldsymbol{\frac{13k+9}{6k+24} = \frac{8}{7}}$

$91k+63 = 48k+192$

$k = 3$

$\therefore$ **The required number is 39.** **[4]**

8. **(a)** **2** **[1]**

(b) Note that $360° - 54° - 90° - 144° = 72°$

The mean of the distribution

$= \frac{2(144)+3(54)+5(72)+7(90)}{360}$

$= \mathbf{4}$ **[2]**

(c) $\frac{72+90}{360} = \boldsymbol{\frac{9}{20}}$

$\therefore$ The required probability is $\frac{9}{20}$. **[2]**

9. **(a)** Since $r_{\text{large}}:r_{\text{small}} = 2:1$, we have $V_{large}:V_{small} = 8:1$

The required volume

$= 324\pi \times \frac{8}{1+8}$

$= \mathbf{288\pi}$ **cm³** **[2]**

(b) $\frac{4}{3}\pi r_{\text{large}}^3 = 288\pi$

$r_{\text{large}} = 6$

$\therefore$ The required radius is 3 cm.

The sum of the surface areas

$= 4\pi 6^2 + 4\pi 3^2$

$= \mathbf{180\pi}$ **cm²** **[3]**

Section A(2)	**[35]**

10. (a) **Let $h(x) = a + bx$, where a and b are non-zero constants.**

$\because h(-2) = -96$ and $h(5) = 72$

$\therefore \begin{cases} a - 2b = -96 \\ a + 5b = 72 \end{cases}$

Solving, we have $a = -48$ and $b = 24$

$\therefore h(x) = 24x - 48$ **[3]**

(b)

$$h(x) = 3x^2$$

$$3x^2 - 24x + 48 = 0$$

$$\boldsymbol{x = 4}$$

[2]

11. (a) Let $ax + b$ be the required quotient.

$p(x) = (ax + b)(2x^2 + 9x + 14)$

$\because p(1) = 50$ and $p(-2) = -52$

$\therefore \begin{cases} (a + b)(2 + 9 + 14) = 50 \\ (-2a + b)(8 - 18 + 14) = -52 \end{cases}$

Solving, we have $\boldsymbol{a = 5}$ **and** $\boldsymbol{b = -3}$**.**

$\therefore$ The required quotient is $5x - 3$. **[3]**

(b)

$$p(x) = 0$$

$$(5x - 3)(2x^2 + 9x + 14) = 0$$

$5x - 3 = 0$ or $2x^2 + 9x + 14 = 0$

For $2x^2 + 9x + 14 = 0$,

$\Delta = 9^2 - 4(2)(14) = -31 < 0$

$\therefore 2x^2 + 9x + 14 = 0$ does not have real roots.

For $5x - 3 = 0$,

$x = \frac{3}{5}$ which is a rational root

$\therefore$ **$p(x) = 0$ has 1 rational root.** **[3]**

12. (a) $72 - (60 + c) = 8$

$$\boldsymbol{c = 4}$$

[2]

(b) **(i)** $(80+b)-(50+a)>34$

$b-a>4$

$\frac{50+a+60(2)+63+64(2)+68+69(3)+70+71(3)+72(2)+75+76+79+80+b}{20}=69$

$a+b=7$

By solving, we have $\begin{cases} \boldsymbol{a=0} \\ \boldsymbol{b=7} \end{cases}$ **or** $\begin{cases} \boldsymbol{a=1} \\ \boldsymbol{b=6} \end{cases}$. **[4]**

(ii) Case 1: $a=0$ and $b=7$

S.D. = 7.582875444

Case 2: $a=1$ and $b=6$

S.D. = 7.341661937

∴ The required S.D. is 7.34 seconds. **[2]**

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13. **(a)** Note that $\angle ABF+\angle AED=180°$

$\angle ABF+115°=180°$

$\angle ABF=65°$

Note that $\angle ABC=90°$

$\angle CBF+65°=90°$

$\boldsymbol{\angle CBF=25°}$ **[3]**

(b) $\angle ODF=\angle CBF=25°$

$\angle OBF=\angle ODF=25°$

$\angle DOF=2\angle CBF=2(25°)=50°$

$\angle BOC$
$=180°-\angle DOF-\angle OBF-\angle ODF$
$=180°-50°-25°-25°$
$=80°$

The perimeter of the sector OBC
$=\frac{80}{360}(2\pi(18))+2(18)$
$=61.13$
>60

∴The required perimeter is not less than 60 cm. **[5]**

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14. **(a)** **(i)**

$BC=BC$	(common side)
$\angle BCG=\angle CBF$	(alt. ∠s, *CG*//*DB*)
$\angle CBG=\angle BCF$	(alt. ∠s, *BG*//*EC*)
$\Delta BCG\cong\Delta CBF$	(ASA) **[2]**

(ii) $\angle CBF = \angle EDF$ (alt. ∠s, $BC // ED$)

$\angle BFC = \angle DFE$ (vert. opp. ∠ s)

$\angle BCF = \angle DEF$ (∠ sum of Δ)

$\Delta BCF \sim \Delta DEF$ (AAA) **[2]**

(b) **(i)** By (a)(i), $\angle BGC = \angle BFC$

$\angle BCF = \angle BFC$

$\therefore BF = BC = l$

$BD \cos 45° = l$

$BD = \sqrt{2}l$

$DF = BD - BF = \sqrt{2}l - l = \boldsymbol{(\sqrt{2} - 1)l}$ **[2]**

(ii) By (b)(i), ΔBCF is an isosceles triangle with $BC = BF$

By (a)(ii), ΔDEF is an isosceles triangle with $DE = DF$

AE

$= AD - DE$

$= AD - DF$

$= l - (\sqrt{2} - 1)l$

$= (2 - \sqrt{2})l$

$> \frac{l}{2}$

$AE + DE = l$

$\therefore DE < \frac{l}{2}$

$DF < \frac{l}{2}$

$\therefore AE > DF$

∴**The claim is agreed.** **[2]**

Section B [35]

15. The required number

$= C_5^{32} - C_5^{11}$

$= \mathbf{200914}$ **[3]**

16. **(a)** Putting $\beta = 5\alpha - 18$ in $\beta = \alpha^2 - 13\alpha + 63$, we have

$5\alpha - 18 = \alpha^2 - 13\alpha + 63$

$\alpha^2 - 18\alpha + 81 = 0$

Solving, we have $\boldsymbol{\alpha = 9}$ **and** $\boldsymbol{\beta = 27}$ **[2]**

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(b) Let $T(n)$ be the nth term of the arithmetic sequence.

$T(1) = \log 9 = 2\log 3$ and $T(2) = \log 27 = 3\log 3$

$d = T(2) - T(1) = \log 3$

$T(1) + T(2) + T(3) + \cdots + T(n) > 888$

$\frac{n}{2}(2(2\log 3) + (n-1)\log 3) > 888$

$(\log 3)n^2 + (3\log 3)n - 1776 > 0$

$n < -62.53$ or $n > 59.53$

$\therefore$ **The least value of n is 60.** **[4]**

17. **(a)** $\frac{r(CD)}{2} + \frac{r(DE)}{2} + \frac{r(CE)}{2} = a$

$r(CD + DE + CE) = 2a$

$\boldsymbol{pr = 2a}$ **[2]**

(b) **(i)** Γ is the angle bisector of $\angle OHK$. **[1]**

(ii) $OH = \sqrt{9^2 + 12^2} = 15$

$HK = \sqrt{(9-14)^2 + 12^2} = 13$

Note that the area of $\Delta OHK = \frac{14(12)}{2} = 84$.

Also note that the perimeter of $\Delta OHK = 13 + 14 + 15 = 42$

Let r be the radius of the inscribed circle of ΔOHK.

By (a), we have $42r = 2(84)$

So, we have $r = 4$

Let $(h, 4)$ be the coordinates of the in-centre of ΔOHK.

$\therefore$ we have $(15 - h) + (14 - h) = 13$

$\therefore h = 8$

The slope of Γ

$= \frac{12-4}{9-8}$

$= 8$

The equation of Γ:

$y - 4 = 8(x - 8)$

$\boldsymbol{8x - y - 60 = 0}$ **[4]**

18. (a) (i) By sine formula, we have

$$\frac{\sin\angle BAD}{BD}=\frac{\sin\angle ABD}{AD}$$

$$\frac{\sin\angle BAD}{12}=\frac{\sin 72°}{13}$$

$\angle BAD = 61.3898°$ or $118.6101°$(rejected)

$\therefore \boldsymbol{\angle BAD = 61.4°}$ **[2]**

(ii) $\angle ADB = 180° - 72° - 61.3898° = 46.6102°$

$$\cos\angle ADB = \frac{AD - AP}{BD}$$

$AP = 13 - 12\cos 46.6102° = 4.75650217$

Note that $\angle CAP = 60°$

By cosine formula, we have

$CP^2 = AC^2 + AP^2 - 2(AC)(AP)\cos\angle CAP$

$CP^2 = 13^2 + 4.76^2 - 2(13)(4.76)\cos 60°$

$\boldsymbol{CP} = 11.39199719 = \mathbf{11.4\ cm}$ **[3]**

(b) $AP^2 + CP^2$

$= 4.76^2 + 11.39^2$

$= 152.4140341$

$AC^2 = 169$

$\because AP^2 + CP^2 \neq AC^2$

$\therefore \angle APC$ is not a right angle.

$\because \angle BPC$ is not the angle between the face ABD and the face ACD.

$\therefore$**The claim is not correct**. **[2]**

19. (a) $f(4)$

$$= \frac{1}{1+k}\left(4^2 + 4(6k-2) + (9k+25)\right)$$

$$= \frac{1}{1+k}(33+33k)$$

$= 33$

$\therefore$The graph of $y = f(x)$ passes through F. **[1]**

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(b) **(i)** $g(x)$

$= f(-x) + 4$

$= \frac{1}{1+k}\left((-x)^2 + (6k-2)(-x) + (9K+25)\right) + 4$

$= \frac{1}{1+k}\left(x^2 - (6k-2)x + (3k-1)^2 - (3k-1)^2 + (9k+25)\right) + 4$

$= \frac{1}{1+k}\left((x-3k+1)^2 - (k+1)(9k-24)\right) + 4$

$= \frac{1}{1+k}\left(x-(3k-1)\right)^2 + (28-9k)$

$\therefore U\ (\mathbf{3k-1, 28-9k})$ **[4]**

(ii) Note that the area of the circle passing through F and O is the least when FO is a diameter of the circle.

If U lies on this circle, then we have $\angle FUO = 90°$

Under this case, we have $k \neq \frac{1}{3}$ and $k \neq \frac{5}{3}$

$$\frac{(28-9k)-0}{(3k-1)-0} \times \frac{33-(28-9k)}{4-(3k-1)} = -1$$

$$\frac{(28-9k)(5+9k)}{(3k-1)(5-3k)} = -1$$

$$2k^2 - 5k - 3 = 0$$

$\boldsymbol{k = 3}$ **or** $\boldsymbol{k = -\frac{1}{2}}$ **(rejected)**

$\therefore$ The area of the circle is the least when $k = 3$. **[4]**

(iii) **Note that** $\boldsymbol{G(-4, 37)}$

$m_{FG} \times m_{GO}$

$= \frac{37-33}{-4-4} \times \frac{37-0}{-4-0}$

$= \frac{37}{8}$

$\neq 1$

$\therefore \angle FGO \neq 90°$

$\because \angle FGO + \angle FVO \neq 180°$

$\therefore$ $\boldsymbol{F, G, O}$ **and** $\boldsymbol{V}$ **are not concyclic.** **[3]**