# HKDSE Mathematics 2018 Core Paper 1 – Suggested Solution

| **Section A(1)** | **[35]** |
|---|---|
| **1.** $\frac{a+4}{3}=\frac{b+1}{2}$<br>$2a+8=3b+3$<br>$3b=2a+5$<br>$\boldsymbol{b=\frac{2a+5}{3}}$ | **[3]** |
| **2.** $\frac{xy^7}{(x^{-2}y^3)^4}$<br>$=\frac{xy^7}{x^{-8}y^{12}}$<br>$=\frac{x^{1+8}}{y^{12-7}}$<br>$=\boldsymbol{\frac{x^9}{y^5}}$ | **[3]** |
| **3.** **(a)** **266** | **[1]** |
| **(b)** **265.4** | **[1]** |
| **(c)** **270** | **[1]** |
| **4.** $\frac{8}{n+5+8}=\frac{2}{5}$<br>$2n+26=40$<br>$\boldsymbol{n=7}$ | **[3]** |
| **5.** **(a)** $9r^3-18r^2s$<br>$=\boldsymbol{9r^2(r-2s)}$ | **[1]** |
| **(b)** $9r^3-18r^2s-rs^2+2s^3$<br>$=9r^2(r-2s)-s^2(r-2s)$<br>$=(r-2s)(9r^2-s^2)$<br>$=\boldsymbol{(r-2s)(3r+s)(3r-s)}$ | **[3]** |
| **6.** **(a)** $\frac{3-x}{2}>2x+7$<br>$3-x>4x+14$<br>$-5x>11$<br>$\boldsymbol{x<-\frac{11}{5}}$<br>$x+8\geq 0$<br>$x\geq -8$<br>$\therefore$ The required range is $\boldsymbol{-8\leq x<-\frac{11}{5}}$ | **[3]** |
| **(b)** $-3$ | **[1]** |

**7.** Let $\$x$ be the cost of the vase.

$\boldsymbol{c - c(1 + 30\%)(1 - 40\%) = 88}$

$0.22c = 88$

$c = 400$

$\therefore$ **The required marked price**

$= \mathbf{400(1 + 30\%)}$

$= \$\mathbf{520}$ **[5]**

**8.** $\boldsymbol{x = 180° - \theta}$

$\angle ADE = x = 180° - \theta$

$\angle BED = x = 180° - \theta$

$y$

$= 180° - \angle ADE - \angle BED$

$= 180° - 2(180° - \theta)$

$= \boldsymbol{2\theta - 180°}$ **[5]**

**9.** Let $x$ minutes be the required time.

$\mathbf{72\left(\frac{x}{60}\right) + 90\left(\frac{161 - x}{60}\right) = 210}$

$18x = 1890$

$\boldsymbol{x = 105}$

$\therefore$The required time is 105 minutes. **[5]**

## Section A(2) [35]

**10.** **(a)** $a - 27 = 21$

$\boldsymbol{a = 48}$

$b - 19 = 43$

$\boldsymbol{b = 62}$ **[3]**

**(b)** Note that $38 - 20 = 18$

$\therefore$ The least possible age of the clerks in term $Y$ is 18.

The greatest possible range of the distribution of the ages of the clerks in the section

$= 62 - 18$

$= 44$

$\neq 43$

$\therefore$ **The claim is disagreed.** **[2]**

**11.** **(a)** **(i)** **1** **[1]**

**(ii)** **8** **[1]**

**(b)** **(i)** **3** **[1]**

**(ii)** **19** **[1]**

**(c)** $\frac{0(k)+1(2)+2(9)+3(6)+4(7)}{k+2+9+6+7} = 2$

$\frac{66}{k+24} = 2$

$\boldsymbol{k = 9}$

**[2]**

**12.** **(a)** $f(3) = 0$

$4(3)(3+1)^2 + a(3) + b = 0$

$3a + b = -192$

$f(-2) = 2b + 1654$

$4(-2)(-2+1)^2 + a(-2) + b = 2b + 165$

$2a + b = -173$

Solving, we have $\boldsymbol{a = -19}$ **and** $\boldsymbol{b = -135}$

**[3]**

**(b)** $f(x) = 0$

$4x(x+1)^2 - 19x - 135 = 0$

$4x^3 + 8x^2 - 15x - 135 = 0$

$(x-3)(4x^2 + 20x + 45) = 0$

$x = 3$ or $4x^2 + 20x + 45 = 0$

For $4x^2 + 20x + 45 = 0$,

$\Delta = 20^2 - 4(4)(45)$

$= -320$

$< 0$

So, the equation $4x^2 + 20x + 45 = 0$ has no real roots.

Note that 3 is not an irrational number.

$\therefore$ **The claim is disagreed.**

**[4]**

**13. (a)**

$\angle ABE = 90°$ (given)

$\angle DCE = 180° - \angle ABE$ (int. ∠ s, AB//DC)

$\angle DCE = 90°$

$\angle ABE = \angle DCE = 90°$

$\angle BAE = 180° - \angle ABE - \angle AEB$ (∠ sum of Δ)

$\angle BAE = 90° - \angle AEB$

$\angle AED = 90°$ (given)

$\angle CED = 180° - \angle AED - \angle AEB$ (adj. ∠ s on st. line)

$\angle CED = 90° - \angle AEB$

$\angle BAE = \angle CED$

$\angle AEB = \angle CDE$ (∠ sum of Δ)

$\Delta ABE \sim \Delta ECD$ (AAA) **[2]**

**(b) (i)**

$BE$

$= \sqrt{AE^2 - AB^2}$

$= \sqrt{25^2 - 15^2}$

$= 20$ cm

$\frac{CD}{BE} = \frac{CE}{AB}$ (By (a))

$\frac{CD}{20} = \frac{36}{15}$

$\boldsymbol{CD = 48}$ **cm** **[2]**

**(ii)**

The area of $\Delta ADE$

$= \frac{1}{2}(AB + CD)(BC) - \frac{1}{2}(AB)(BE) - \frac{1}{2}(CD)(CE)$

$= \frac{1}{2}(15 + 48)(20 + 36) - \frac{1}{2}(15)(20) - \frac{1}{2}(48)(36)$

$= \boldsymbol{750\ cm^2}$ **[2]**

**(iii)**

$AD$

$= \sqrt{BC^2 + (CD - AB)^2}$

$= \sqrt{(20 + 36)^2 + (48 - 15)^2}$

$= 65$ cm

The shortest distance from $E$ to $AD$

$= \frac{2(750)}{65}$

$= 23.077$

$> 23$

∴ **There is no such point *F*.** **[2]**

**14.** **(a)** The required volume

$= \pi(8^2)(64)$

$= \mathbf{4096\pi}$ **cm³** **[2]**

**(b)** Let $h$ cm be the depth of water in the vessel.

$\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = 4096\pi$

$h^3 = 110592$

$h = 48$

$\therefore$ The required depth is 48 cm. **[4]**

**(c)** The volume not occupied by water in the vessel

$= \frac{1}{3}\pi(20^2)(60) - 4096\pi$

$= 3904\pi$ cm$^3$

The volume of the metal sphere

$= \frac{4}{3}\pi(14^3)$

$= \frac{10976}{3}\pi$ cm$^3$

$< 3904\pi$ cm$^3$

$\therefore$ **The water will not overflow.** **[3]**

## Section B [35]

**15.** **(a)** The required number

$= 8!$

$= \mathbf{40320}$ **[1]**

**(b)** The required number

$= P_2^4 \times P_6^6$

$= \mathbf{8640}$ **[2]**

**16.** **(a)** Let $a$ and $r$ be the first term and the common ratio of the sequence respectively.

$\begin{cases} ar^2 = 720 \\ ar^3 = 864 \end{cases}$

Solving, we have $a = 500$

$\therefore$ **The first term is 500.** **[2]**

**(b)** Note that $r = 1.2$

$500(1.2^n) + 500(1.2^{2n}) < 5 \times 10^{14}$

$(1.2^n)^2 + (1.2^n) - 10^{12} < 0$

$\frac{-1-\sqrt{1^2-4(1)(-10^{12})}}{2} < 1.2^n < \frac{-1+\sqrt{1^2-4(1)(-10^{12})}}{2}$

$\log 1.2^n < \log\left(\frac{-1+\sqrt{1^2-4(1)(-10^{12})}}{2}\right)$

$n < 75.7755$

$\therefore$ **The greatest value of $n$ is 75.** **[3]**

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**17. (a)** By sine formula, we have

$\frac{AD}{\sin 20°} = \frac{60}{\sin(180°-120°-20°)}$

$AD = 31.92533317$ cm

$\boldsymbol{AD = 31.9}$ **cm** **[2]**

**(b) (i)** By cosine formula, we have

$\cos\angle ABC = \frac{AB^2+BC^2-AC^2}{2(AB)(BC)}$

$\cos\angle ABC = \frac{60^2+(31.92633317)^2-40^2}{2(60)(31.92633317)}$

$\angle ABC = 37.99207534°$

$\boldsymbol{\angle ABC = 38.0°}$ **[2]**

**(ii)** In Figure 3(a), $AP$ produced meets $CD$ at $Q$,

where $P$ is the foot of the perpendicular from $A$ to $BD$ .

Note that the required angle is $\angle APQ$ in Figure 3(b).

$AP$

= $AD \sin\angle ADP$

= 31.92533317 sin(180° -120° - 20°)

= 20.5212086 cm

$DP^2 = AD^2 - AP^2$

$DP^2 = (31.92533317)^2 - (20.5212086)^2$

$DP = 24.45622407$ cm

$PQ$

= $DP \tan\angle PDQ$

= (24.45622407) tan 20°

= 8.901337605 cm

$DQ^2 = DP^2 + PQ^2$

$DQ^2 = (24.45622407)^2 + (8.901337605)^2$

$DQ = 26.02577006$ cm

Note that $\angle ADC = \angle ABC = 37.99207534°$ .

By cosine formula, we have

$AQ^2 = AD^2 + DQ^2 - 2(AD)(DQ)\cos\angle ADC$

$AQ = 9.67076991$ cm

By cosine formula, we have

$$\cos\angle APQ = \frac{AP^2 + PQ^2 - AQ^2}{2(AP)(PQ)}$$

$\angle APQ = 71.91411397°$

$\angle APQ = 71.9°$

$\therefore$ **The required angle is 71.9°.** **[3]**

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**18.** **(a)** **Let $f(x) = ax^2 + bx$**

$\therefore \begin{cases} 4a + 2b = 60 \\ 9a + 3b = 99 \end{cases}$

Solving, we **have $a = 3$ and $b = 24$ .**

$\therefore\ f(x) = 3x^2 + 24x$ **[3]**

**(b)** **(i)** $f(x)$

$= 3x^2 + 24x$

$= 3(x^2 + 8x)$

$= 3(x^2 + 8x + 16 - 16)$

$= 3(x + 4)^2 - 48$

$\therefore$ $\boldsymbol{Q\ (-4, -48)}$ . **[2]**

**(ii)** $(-4, 75)$ **[1]**

**(iii)** $m_{QS} = \frac{-48-0}{-4-56}$

$= \frac{4}{5}$

$m_{RS} = \frac{75-0}{-4-56}$

$= -\frac{5}{4}$

$\therefore m_{QS} \times m_{RS} = -1$

$\therefore$ $\angle QSR$ is a right angle.

$\therefore$ $QR$ is a diameter of the circumcircle of $\Delta QRS$ .

Note that $P$ is the circumcentre of $\Delta QRS$ .

$\therefore$ **$P$ is the mid-point of the line segment joining $Q$ and $R$.** **[2]**

**19. (a)** **The equation of $C$ is $(x-8)^2+(y-2)^2=r^2$**

Putting $y=\frac{kx-21}{5}$ in $(x-8)^2+(y-2)^2=r^2$, we have

$$(x-8)^2+\left(\frac{kx-21}{5}-2\right)^2=r^2$$

$(k^2+25)x^2+(-62k-400)x+2561-25r^2=0$

Note that $L$ is a tangent to $C$.

$\Delta=(-62k-400)^2-4(k^2+25)(2561-25r^2)=0$

$$r^2=\frac{64k^2-496k+961}{k^2+25}$$

**[4]**

**(b) (i)** $\because$ $L$ passes through $D$

$\therefore$ $18k-5(39)-21=0.$

$k=12.$

By (a), we have $r^2=\frac{64(12)^2-496(12)+961}{(12)^2+25}$

$r=5$

**[3]**

**(ii)** Let $G$ be the centre of $C$ .

Note that $E(0,-\frac{21}{5})$ .

Also note that $G$ is the in-centre of $\Delta DEF$ .

$DG^2=(18-8)^2+(39-2)^2$

$DG=\sqrt{1469}$

$\sin\angle EDG=\frac{r}{DG}$

$\sin\angle EDG=\frac{5}{\sqrt{1469}}$

$\angle EDG=7.49585764°$

$EG^2=(8-0)^2+\left(2+\frac{21}{5}\right)^2$

$EG=\frac{\sqrt{2561}}{5}$

$\sin\angle DEG=\frac{r}{EG}$

$\sin\angle DEG=\frac{25}{\sqrt{2561}}$

$\angle DEG=29.60445074°$

Note that $\angle EDG=\angle FDG$ and $\angle DEG=\angle FEG$ .

$\angle DFE$

$=180°-(\angle EDG+\angle FDG)-(\angle DEG+\angle FEG)$

$=180°-2(7.49585764°)-2(29.60445074°)$

$=105.7\,993832°$

**[5]**

$> 90°$

$\therefore$ **$\Delta DEF$ is an obtuse-angled triangle.**