HKDSE Mathematics 2017 Core Paper 1 – Suggested Solution

Section A(1)	**[35]**
1. $k = \frac{3x-y}{y}$ $ky = 3x - y$ $y(k+1) = 3x$ $\boldsymbol{y = \frac{3x}{k+1}}$	**[3]**
2. $\frac{(m^4n^{-1})^3}{(m^{-2})^5}$ $= \frac{m^{12}n^{-3}}{m^{-10}}$ $= \frac{m^{12+10}}{n^3}$ $= \boldsymbol{\frac{m^{22}}{n^3}}$	**[3]**
3. (a) $x^2 - 4xy + 3y^2$ $= \boldsymbol{(x-3y)(x-y)}$	**[1]**
(b) $x^2 - 4xy + 3y^2 + 11x - 33y$ $= (x-3y)(x-y) + 11(x-3y)$ $= \boldsymbol{(x-3y)(x-y+11)}$	**[2]**
4. **Let x and $5x$ be the number of concessionary tickets and regular tickets sold respectively.** $\boldsymbol{5x(126) + x(78) = 50976}$ $708x = 50976$ $x = 72$ The total number of admission tickets sold that day $= 6(72)$ $= 432$ $\therefore$ **The total number of admission tickets sold that day is 432.**	**[4]**
5. (a) $7(x-2) \le \frac{11x+8}{3}$ $21x - 42 \le 11x + 8$ $\boldsymbol{x \le 5}$ Also, $6 - x < 5$ $\boldsymbol{x > 1}$ $\therefore$ the required range is $1 < x \le 5$.	**[3]**
(b) **4**	**[1]**

6. **(a)** $\boldsymbol{A'\ (-4, -3)}$

$\boldsymbol{B'\ (9, 9)}$ **[2]**

(b) m_{AB}

$= \frac{4+9}{-3-9}$

$= -\frac{13}{12}$

$m_{A'B'}$

$= \frac{-3-9}{-4-9}$

$= \frac{12}{13}$

$\therefore m_{AB} \times m_{A'B'} = -1$

$\therefore$ **AB is perpendicular to $A'B'$** **[2]**

7. **(a)** $\frac{x}{360} = \frac{1}{9}$

$x = \frac{360}{9}$

$\boldsymbol{x = 40°}$ **[2]**

(b) Let n be the number of students in the school.

$\frac{n}{360} = \frac{180}{360 - 90 - 158 - 40}$

$\frac{n}{360} = \frac{180}{72}$

$n = 900$

$\therefore$ **the number of students in the school is 900 .** **[2]**

8. **(a)** **Let $\boldsymbol{y = \frac{k}{\sqrt{x}}}$**

So, we have $81 = \frac{k}{\sqrt{144}}$

Solving, we have **k = 972**

$\therefore\ y = \frac{972}{\sqrt{x}}$ **[3]**

(b) Note that the initial value of y is 81 .

The final value of y

$= \frac{972}{\sqrt{324}}$

$= 54$

The decrease in the value of y

$= 81 - 54$

$= \mathbf{27}$ **[2]**

9. **(a)** The maximum absolute error

$= 5$ mL

The least possible capacity

$= 200 - 5$

$= \mathbf{195\ mL}$ **[2]**

(b) The least possible total capacity of 120 standard bottles

$= (195)(120)$

$= \mathbf{23400mL}$

$= 23.4$L

> 23.35 L

∴ **the claim is disagreed** **[3]**

Section A(2) **[35]**

10. **(a)**

$OP = OR$	(given)
$PS = RS$	(given)
$OS = OS$	(common side)
$\Delta OPS \cong \Delta ORS$	(SSS)

[2]

(b) $\angle POQ$

$= 2\angle PRQ$

$= 2(10°)$

$= 20°$

$\angle POR$

$= \angle POQ + \angle QOR$

$= \angle POQ + \angle POQ$ (by (a))

$= 2\angle POQ$

$= 40°$

The area of the sector OPQR

$= \dfrac{40}{360}(6^2)\pi$

$= \mathbf{4\pi\ cm^2}$ **[4]**

11. (a) The median

$= \$69$

Note that $\frac{61(3)+63+64+66+68+69(2)+70+a+77(2)+78+81+80+b}{15} = 70$

$a + b = 5$

Also note that $(80 + b) - 61 = 22$

Solving, we have $\boldsymbol{a = 2}$ **and** $\boldsymbol{b = 3}$

The standard deviation

$= \$\mathbf{7.33}$ **[5]**

(b) The required probability

$= \frac{6}{15}$

$= \frac{2}{5}$ **[2]**

12. (a) Let V cm³ be the volume of the larger pyramid.

$V + V \times \left(\sqrt{\frac{4}{9}}\right)^3 = (84)(20)$

$\frac{35V}{27} = 1680$

$\boldsymbol{V = 1296}$

∴ The volume of the larger pyramid is 1296 cm³ **[3]**

(b) The length of the side of the base of the larger pyramid

$= \sqrt{\frac{3(1296)}{12}}$

$= 18$cm

The total surface area of the larger pyramid

$= (18)(18) + 4\left(\frac{1}{2}(18)\sqrt{\left(\frac{18}{2}\right)^2 + 12^2}\right)$

$= 864$ cm²

The total surface area of the smaller pyramid

$= \frac{4}{9}(864)$

$= \mathbf{384}$ **cm²**

∴ The total surface area of the smaller pyramid is 384 cm² **[4]**

13. (a) The equation of C is

$(x-2)^2 + (y+1)^2 = (-6-2)^2 + (5+1)^2$

$\boldsymbol{(x-2)^2 + (y+1)^2 = 100}$ **[2]**

(b) Note that the radius of C is 10 .

FG

$= \sqrt{(-3-2)^2 + (11+1)^2}$

$= 13$

> 10

$\therefore$ ***F* lies outside *C* .** **[2]**

(c) **(i)** F, G and H are collinear. **[1]**

(ii) m_{FH}

$= m_{FG}$

$= \frac{11-(-1)}{-3-2}$

$= -\frac{12}{5}$

The required equation is

$y - 11 = -\frac{12}{5}(x - (-3))$

$5y - 55 = -12x - 36$

$\mathbf{12x + 5y - 19 = 0}$ **[2]**

---

14. **(a)** Note that $f(x) = (2x^2 + ax + 4)(3x + 7) + bx + c$.

$= 6x^3 + 3ax^2 + 12x + 14x^2 + 7ax + 28 + bx + c$.

Also note that $f(x) = 6x^3 - 13x^2 - 46x + 34$.

By comparing coefficient, we have $3a + 14 = -13$

$\boldsymbol{a = -9}$ **[3]**

(b) **(i)** Let $g(x) = k(2x^2 + ax + 4) + bx + c$, where k is a constant.

$f(x) - g(x)$

$= (2x^2 + ax + 4)(3x + 7) - k(2x^2 + ax + 4)$

$= (2x^2 + ax + 4)(3x + 7 - k)$

$\therefore$ $\boldsymbol{f(x) - g(x)}$ **is divisible by** $\boldsymbol{2x^2 + ax + 4}$ **.** **[2]**

(ii) $f(x) - g(x) = 0$

$(2x^2 - 9x + 4)(3x + 7 - k) = 0$ (by (a) and (b)(i))

$(2x - 1)(x - 4)(3x + 7 - k) = 0$

Note that $\frac{1}{2}$ is a root of the above equation.

$\therefore$ Not all the roots of the equation $f(x) - g(x) = 0$ are integers.

$\therefore$ **The claim is disagreed.** **[3]**

Section B [35]

15.
$\begin{cases} 3 = a + \log_b 243 \\ 0 = a + \log_b 9 \end{cases}$

$\begin{cases} b^{3-a} = 243 \\ b^{-a} = 9 \end{cases}$

$\begin{cases} 9(b^3) = 243 \\ b^{-a} = 9 \end{cases}$

Solving, we have $\boldsymbol{a = -2}$ **and** $\boldsymbol{b = 3}$

$y = -2 + \log_3 x$

$y + 2 = \log_3 x$

$\boldsymbol{x = 3^{y+2}}$ **[4]**

16. (a) The required volume

$= 1.5 \times 10^7 + 1.5 \times 10^7 \times 0.9 + 1.5 \times 10^7 \times (0.9)^2 + \cdots + 1.5 \times 10^7 \times (0.9)^{19}$

$= \frac{1.5 \times 10^7 (1 - 0.9^{20})}{1 - 0.9}$

$= \boldsymbol{1.32 \times 10^8 \text{ m}^3}$ **[2]**

(b) The total volume of water imported since the start of the plan

$< 1.5 \times 10^7 + 1.5 \times 10^7 \times 0.9 + 1.5 \times 10^7 \times (0.9)^2 + \cdots$

$= \frac{1.5 \times 10^7}{1 - 0.9}$

$= 1.5 \times 10^8 \text{ m}^3$

$< 1.6 \times 10^8 \text{ m}^3$

$\therefore$ **The claim is agreed.** **[2]**

17. (a) The required probability

$= \frac{C_4^4 C_1^{15}}{C_5^{19}}$

$= \frac{5}{3876}$ **[2]**

(b) The required probability

$= \frac{C_3^4 C_2^{15}}{C_5^{19}}$

$= \frac{35}{969}$ **[2]**

(c) The required probability

$= 1 - \frac{5}{3876} - \frac{35}{969}$

$= \frac{3731}{3876}$ **[2]**

18. **(a)** Putting $y = 19$ in $y = 2x^2 - 2kx + 2x - 3k + 8$, we have

$$2x^2 - 2kx + 2x - 3k + 8 = 19.$$
$$2x^2 + (2 - 2k)x - (3k + 11) = 0.$$

Δ

$= (2 - 2k)^2 - 4(2)(-(3k + 11))$

$= 4k^2 + 16k + 92$

$= 4(k^2 + 4k + 2^2 - 2^2) + 92$

$= 4(k + 2)^2 + 76$

> 0

$\therefore$ ***L* and *Γ* intersect at two distinct points.** **[3]**

(b) **(i)** Note that the roots of the equation $2x^2 + (2 - 2k)x - (3k + 11) = 0$ are a and b.

$\therefore a + b = -\frac{2-2k}{2} = k - 1$ and $ab = -\frac{3k+11}{2}$

$(a - b)^2$

$= a^2 - 2ab + b^2$

$= (a + b)^2 - 4ab$

$= (k - 1)^2 - 4\left(-\dfrac{3k + 11}{2}\right)$

$= k^2 - 2k + 1 + 6k + 22$

$= \boldsymbol{k^2 + 4k + 23}$ **[3]**

(ii) $AB < 4$

$\sqrt{(a - b)^2} < 4$

$(a - b)^2 < 16$

$k^2 + 4k + 23 < 16$

$k^2 + 4k + 7 < 0$

Note that $42 - 4(1)(7) = -12 < 0$.
Also note that the coefficient of k^2 is positive.

$\therefore k^2 + 4k + 7 < 0$ has no solutions.

$\therefore$ **It is not possible.** **[2]**

19. **(a)** By sine formula, we have

$$\frac{AC}{\sin\angle ABC}=\frac{BC}{\sin\angle BAC}$$

$$\frac{AC}{\sin(180°-30°-42°)}=\frac{24}{\sin 30°}$$

$$AC=45.65071278\text{ cm}$$

$$\boldsymbol{AC=45.7}\textbf{ cm}$$

∴ The length of AC is 45.7 cm . **[2]**

(b) **(i)** Since $\Delta ADF\sim\Delta CEF$, we have $\frac{AF}{CF}=\frac{AD}{CE}$

$$\frac{AF}{CF}=\frac{10}{2}$$

$$AC+CF=5CF$$

$$CF=\frac{1}{4}AC$$

$$CF=11.4126782\text{ cm}$$

$$\boldsymbol{CF=11.4}\textbf{ cm}$$

∴ The distance between C and F is 11.4 cm . **[2]**

(ii) The area of ΔABF

$$=\frac{1}{2}(AC+CF)(BC\sin\angle ACB)$$

$$=\frac{1}{2}(45.65071278+11.4126782)(24\sin 42°)$$

$$=458.1943369$$

$$=\mathbf{458\text{ cm}^2}$$ **[2]**

(iii) Note that $\angle BCF=138°$.

By cosine formula, we have

$$BF^2=CF^2+BC^2-2(BC)(BF)\cos\angle BCF$$

$$BF^2=(11.4126782)^2+24^2-2(11.4126782)(24)\cos 138°$$

$$BF=33.36690449\text{ cm}$$

Let *P* be the foot of the perpendicular from *A* to *BF* produced.

Note that the area of $\Delta ABF=\frac{1}{2}(AP)(BF)$.

AP

$$=\frac{2(458.1943369)}{33.36690449}$$

$$=27.46400026\text{ cm}$$

Since DP is perpendicular to FB produced, the inclination of the thin metal sheet ABC to the horizontal ground is $\angle APD$.

$\sin \angle APD = \dfrac{AD}{AP}$

$\sin \angle APD = \dfrac{10}{27.46400026}$

$\angle APD = 21.35300646°$

∴ **The required inclination is 21.4°**

[5]

(iv) The area of ΔBDF

$= (\text{The area of } \Delta ABF)\cos \angle APD$

$\leq \text{The area of } \Delta ABF$

$= 458 \text{ cm}^2$ (by (b)(ii))

$< 460 \text{ cm}^2$

∴ **The claim is disagreed.** **[2]**