HKDSE Mathematics 2016 Core Paper 1 – Suggested Solution

Section A(1)	**[35]**
1. $\dfrac{(x^8y^7)^2}{x^5y^{-6}}$ $= \dfrac{x^{16}y^{14}}{x^5y^{-6}}$ $= x^{16-5}y^{14-(-6)}$ $= \boldsymbol{x^{11}y^{20}}$	**[3]**
2. $Ax = (4x + B)C$ $Ax = 4Cx + BC$ $Ax - 4Cx = BC$ $(A - 4C)x = BC$ $\boldsymbol{x = \dfrac{BC}{A - 4C}}$	**[3]**
3. $\dfrac{2}{4x-5} + \dfrac{3}{1-6x}$ $= \dfrac{2(1-6x) + 3(4x-5)}{(4x-5)(1-6x)}$ $= \dfrac{2 - 12x + 12x - 15}{(4x-5)(1-6x)}$ $= -\dfrac{\boldsymbol{13}}{\boldsymbol{(4x-5)(6x-1)}}$ $= \dfrac{13}{(4x-5)(6x-1)}$	**[3]**
4. (a) $5m - 10n$ $= \boldsymbol{5(m - 2n)}$	**[1]**
(b) $m^2 + mn - 6n^2$ $= \boldsymbol{(m + 3n)(m - 2n)}$	**[1]**
(c) $m^2 + mn - 6n^2 - 5m + 10n$ $= (m + 3n)(m - 2n) - 5(m - 2n)$ $= \boldsymbol{(m - 2n)(m + 3n - 5)}$	**[2]**
5. **Let $1.4y$ and y be the number of male members and female members respectively.** $\boldsymbol{1.4y + y = 180}$ $y = 75$ $\therefore$ The required difference $= 1.4(75) - 75$ $= \boldsymbol{30}$	**[4]**

6. **(a)** $x+6<6(x+1)$
$x+6<6x+66$
$-5x<60$
$\boldsymbol{x>-12}$
$\therefore x>-12$ or $x\leq -5$

$\therefore$ **The solution is all real numbers.** **[3]**

(b) $\boldsymbol{-1}$ **[1]**

7. **(a)** $\angle AOB$
$=135°-75°$
$=\boldsymbol{60°}$ **[1]**

(b) Since $AO=BO$, we have $\angle OAB=\angle OBA$

Note that $\angle OAB+\angle OBA+60°=180°$
$\therefore\ \angle OAB=\angle OBA=60°$

$\therefore\ \Delta AOB$ is an equilateral triangle.

The required perimeter

$=3(12)$

$=\boldsymbol{36}$ **[2]**

(c) **3** **[1]**

8. **(a)** **Let $\boldsymbol{f(x)=hx+kx^2}$**

$\therefore\begin{cases}3h+9k=48\\9h+81k=198\end{cases}$

Solving, we have $\boldsymbol{h=13}$ **and** $\boldsymbol{k=1}$

$\therefore\ f(x)=13x+x^2$ **[3]**

(b) $f(x)=90$
$13x+x^2=90$
$x^2+13x-90=0$
$(x-5)(x+18)=0$
$\boldsymbol{x=5}$ **or** $\boldsymbol{x=-18}$ **[2]**

9. **(a)** $x=2+4=\boldsymbol{6}$

$y=37-15=\boldsymbol{22}$

$z=37+3=\boldsymbol{40}$ **[3]**

(b) The required probability

$=\dfrac{22-6}{40}$

$=\boldsymbol{\dfrac{2}{5}}$ **[2]**

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Section A(2)	[35]

10. (a) Let (x, y) be the coordinate of P.

$\sqrt{(x-5)^2+(y-7)^2} = \sqrt{(x-13)^2+(y-1)^2}$

$\mathbf{4x - 3y - 24 = 0}$

$\therefore$ The equation of Γ is $4x - 3y - 24 = 0$ **[2]**

(b) Put $y = 0$ into $4x - 3y - 24 = 0$, we have $x = 6$

$\therefore H\ (6, 0)$

Put $x = 0$ into $4x - 3y - 24 = 0$, we have $y = -8$

$\therefore K\ (0, -8)$

Diameter of C

$= HK$

$= \sqrt{(6-0)^2 + (0-(-8))^2}$

$= 10$

Circumference of C

$= 10\pi$

$= 31.416$

> 30

$\therefore$ **The claim is agreed.** **[3]**

11. (a) Let V cm^3 be the required volume.

$\dfrac{V - 444\pi}{V} = \left(\dfrac{12}{16}\right)^3$

$\boldsymbol{V = 768\pi}$

$\therefore$ The required volume is 768π cm^3 **[3]**

(b) Let r cm be the radius of the wet curved surface.

$\dfrac{1}{3}\pi r^2(16) = 768\pi$

$r = 12$

The final area of the we curved surface

$= \pi(12)\sqrt{12^2 + 16^2}$

$= 753.9822369$

< 800 cm^2

$\therefore$ **The claim is disagreed.** **[3]**

12. **(a)** $11 + a = 11 + b + 4$

$a = b + 4$

Note that $a > 11$ and $4 < b < 10$

$\therefore \begin{cases} \boldsymbol{a = 12} \\ \boldsymbol{b = 8} \end{cases}$ **or** $\begin{cases} \boldsymbol{a = 13} \\ \boldsymbol{b = 9} \end{cases}$ **[3]**

(b) **(i)** When the ages of the children are 7, 8, 9 and 10,

the median is the greatest.

The greatest possible median $=$ **8** **[2]**

(ii) When the ages of the children are 6, 7, 8 and 9, the mean is the smallest.

By (a), there are 2 cases.

Case 1: a = 12 and b = 8

The mean = $\frac{12(6)+13(7)+12(8)+9(9)+4(10)}{12+13+12+9+4} = 7.6$

Case 2: a = 13 and b = 9

The mean = $\frac{12(6)+14(7)+12(8)+10(9)+4(10)}{12+14+12+10+4} = 7.61538$

$\therefore$ **The least possible mean is 7.6** **[2]**

13. **(a)** In ΔACD and ΔABE,

$\angle ADC = \angle AEB$ (given)

$AD = AE$ (side opp. equal ∠ s)

$CE = BD$ (given)

$CE + DE = BD + DE$

$CD = BE$

$\Delta ACD \cong \Delta ABE$ (SAS) **[2]**

(b) **(i)** Note that $DM = EM = 9$ cm and $\angle AMD = \angle AME = 90°$

AM

$= \sqrt{AD^2 - DM^2}$

$= \sqrt{144}$

$=$ **12 cm** **[2]**

(ii) AB^2
$= AM^2 + BM^2$
$= 144 + 16^2$
$= 400$

By (a), we have $AE = AD = 15$ cm

$AB^2 + AE^2$
$= 400 + 15^2$
$= 625$

BE^2
$= (BD + DE)^2$
$= (7 + 18)^2$
$= 625$

$\because AB^2 + AE^2 = BE^2$

$\therefore$ **ΔABE is a right-angled triangle.** [3]

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14. (a) Note that $p(2) = 152 + 4a + 2b + c$ and $p(-2) = 40 + 4a - 2b + c$

Since $p(2) = p(-2)$, we have $b = -28$

$(3x^2 + 5x + 8)(2x^2 + mx + n)$

$= 6x^4 + (3m + 10)x^3 + (3n + 5m + 16)x^2 + (8m + 5n)x + 8n$

By comparing the coefficient of each term, we have $\boldsymbol{l = 3}$

$3m + 10 = 7$ and $8m + 5n = -28$

Solving, we have **$m = -1$ and $n = -4$** [5]

(b) $p(x) = 0$
$(3x^2 + 5x + 8)(2x^2 - x - 4) = 0$
$3x^2 + 5x + 8 = 0$ and $2x^2 - x - 4 = 0$

$\Delta = 5^2 - 4(3)(8)$
$= \mathbf{-71}$
< 0
$\therefore$ **$3x^2 + 5x + 8 = 0$ has no real roots.**

$\Delta = (-1)^2 - 4(2)(-4)$
$= 33 > 0$
$\therefore 2x^2 - x - 4 = 0$ has 2 unequal real roots.

$\therefore p(x)$ has **2 unequal real roots**. [5]

Section B [35]

15. The required probability

$= \dfrac{C_4^6 4!\,5!}{(4+5)!}$

$= \mathbf{\dfrac{5}{42}}$ [3]

16. Let σ marks be the standard derivation

$$\frac{22-61}{\sigma} = -2.6$$

$$\sigma = 15$$

Score of Mary

$= 61 + 1.4\sigma$

$=$ **82 marks**

Difference between the score of Mary and that of Albert

$= 82 - 22$

$= 60$

> 59

Note that the range of the distribution must be at least 60.

$\therefore$ **The claim is disagreed.** **[3]**

17. (a) Let d be the common difference of the sequence.

$$555 = 666 + (38-1)d$$

$$\boldsymbol{d = -3}$$ **[2]**

(b) $$\frac{\boldsymbol{n}}{\boldsymbol{2}}\left(\boldsymbol{2(666) + (n-1)(-3)}\right) > \boldsymbol{0}$$

$$1335n - 3n^2 > 0$$

$$0 < n < 445$$

$\therefore$ **The greatest value of n is 444.** **[3]**

18. (a) $f(x)$

$= -\frac{1}{3}x^2 + 12x - 121$

$= -\frac{1}{3}(x^2 - 36x + 18^2 - 18^2) - 121$

$= -\frac{1}{3}(x-18)^2 - 13$

$\therefore$ **The coordinates of the vertex is (18,-13)** **[2]**

(b) $g(x)$

$= f(x) + 13$

$= -\boldsymbol{\frac{1}{3}(x-18)^2}$ **[2]**

(c) Note that $-\frac{1}{3}x^2 - 12x - 121 = f(-x)$

$\therefore$ **$f(x)$ is reflected along the y-axis.** **[2]**

19. (a) By sine formula, we have

$$\frac{AB}{\sin \angle ADB} = \frac{BD}{\sin \angle BAD}$$

$\angle ADB = 41.68560132°$ or $138.3143987°$ (rejected)

$\angle ABD = 180° - \angle BAD - \angle ADB$

$= 52.31439868°$

$= \mathbf{52.3°}$

By cosine formula, we have

$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos \angle CBD$

$CD^2 = 8^2 + 15^2 - 2(8)(15)cos43°$

$CD = 10.65246974$

$\boldsymbol{CD} = \mathbf{10.7\ cm}$

[4]

(b) Since $AC^2 + BC^2 = AB^2$,

we have $\angle ACE = 90°$.

By cosine formula, we have

$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos \angle ABD$

$AD^2 = 10^2 + 15^2 - 2(10)(15)\cos 52.31439868°$

$AD = 11.89964475$

By cosine formula, we have

$$AD^2 = AC^2 + CD^2 - 2(AC)(CD)\cos \angle ACD$$

$$\cos \angle ACD = \frac{6^2 + (10.65246974)^2 - (11.89964475)^2}{2(6)(10.65246794)}$$

$\angle ACD = 86.46867599°$

$\therefore \angle ACD$ is not a right angle.

$\therefore$ The angle between AB and the face BCD is not $\angle ABC$.

∴ The claim is disagreed.

[2]

20. **(a)** Note that J is the center of circle OPQ

$\angle IPO = \angle IPQ$ (incenter)

Also note that P, I and J are collinear.

$\angle JPO = \angle JPQ$

$JO = JP$ (radii)

$\angle JOP = \angle JPO$ (base ∠ s, isos. Δ)

$JP = JQ$ (radii)

$\angle JPQ = \angle JQP$ (base ∠ s, isos. Δ)

$\angle JOP = \angle JQP$

$JP = JP$ (common sides)

$\Delta JOP \cong \Delta JQP$ (AAS)

$\therefore OP = PQ$ (corr. sides, ≅Δ) **[3]**

(b) **(i)** Let $(h, 19)$ be the coordinate of P.

By (a), we have $h^2 + 19^2 = (40 - h)^2 + (30 - 19)^2$

Solving, we have $h = 17$

Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of C

Put (0,0), we have $\boldsymbol{F = 0}$

Put (40, 30) and (17, 19) into C, we have

$17D + 19E + 650 = 0$ and $40D + 30E + 2500 = 0$

Solving, we have $\boldsymbol{D = -112}$ **and** $\boldsymbol{E = 66}$

$\therefore$ The equation of C is $x^2 + y^2 - 112x + 66y = 0$ **[4]**

(ii) Note that the equations of L_1 and L_2 are in the form of $y = \frac{3}{4}x + c$

Put $y = \frac{3}{4}x + c$ into C, we have

$$x^2 + \left(\frac{3}{4}x + c\right)^2 - 112x + 66\left(\frac{3}{4}x + c\right) = 0$$

$$25x^2 + (24c - 1000)x + 16c^2 + 1056c = 0$$

Since L_1 and L_2 are the tangent of C, we have

$$\Delta = (24c - 1000)^2 - 4(25)(16c^2 + 1056c) = 0$$

$c = \frac{25}{4}$ or $-\frac{625}{4}$

So, the equation of L_1 and L_2 are $y = \frac{3}{4}x + \frac{25}{4}$ and $y = \frac{3}{4}x - \frac{625}{4}$ respectively.

Note that the coordinates of *S*, *T*, *U* and *V* are $\left(-\frac{25}{3}, 0\right)$, $\left(0, \frac{25}{4}\right)$, $\left(\frac{625}{4}, 0\right)$ and $\left(0, -\frac{625}{4}\right)$ respectively.

The area of trapezium *STUV*

$= \frac{1}{2}\left(\left(\frac{625}{3}\right)\left(\frac{625}{4}\right) + \left(\frac{625}{4}\right)\left(\frac{25}{3}\right) + \left(\frac{25}{3}\right)\left(\frac{25}{4}\right) + \left(\frac{25}{4}\right)\left(\frac{625}{3}\right)\right)$

$= \frac{105625}{6}$

$= 17604.16666$

> 17000

$\therefore$ **The claim is agreed.** **[5]**