# HKDSE Mathematics 2015 Core Paper 1–Suggested Solution

| Section A(1) | [35] |
|---|---|
| **1.** $\dfrac{m^9}{(m^3n^{-7})^5}$ <br> $=\dfrac{m^9}{m^{15}n^{-35}}$ <br> $=\dfrac{n^{35}}{m^{15-9}}$ <br> $=\boldsymbol{\dfrac{n^{35}}{m^6}}$ | **[3]** |
| **2.** $\dfrac{4a+5b-7}{b}=8$ <br> $4a+5b-7=8b$ <br> $-3b=7-4a$ <br> $\boldsymbol{b=\dfrac{4a-7}{3}}$ | **[3]** |
| **3.** The required probability <br> $=\dfrac{3+2+1}{4\times 5}$ <br> $=\boldsymbol{\dfrac{3}{10}}$ | **[3]** |
| **4.** **(a)** $x^3+x^2y-7x^2$ <br> $=\boldsymbol{x^2(x+y-7)}$ | **[1]** |
| **(b)** $x^3+x^2y-7x^2-x-y+7$ <br> $=x^2(x+y-7)-(x+y-7)$ <br> $=(x+y-7)(x^2-1)$ <br> $=\boldsymbol{(x-1)(x+1)(x+y-7)}$ | **[3]** |
| **5.** **(a)** $\dfrac{(7-3x)}{5}\le 2(x+2)$ <br> $7-3x\le 10x+20$ <br> $\boldsymbol{x\ge -1}$ <br> $4x-13>0$ <br> $\boldsymbol{x>\dfrac{13}{4}}$ <br> $\therefore$ the required solution is $x>\frac{13}{4}$ | **[3]** |
| **(b)** **4** | **[1]** |
| **6.** **(a)** The selling price of the book. <br> $=250(1+20\%)$ <br> $=\$300$ <br> $\therefore$**The selling price of the book is $300** | **[2]** |

**(b)** Let $\$x$ be the marked price of the book.

$(1-25\%)x = 300$

$x = 400$

$\therefore$ **The marked price of the book is $400** **[2]**

**7.** Let $a$ and $b$ be the number of apples owned by Ada and Billy respectively.

$$\begin{cases} a = 4b \\ a - 12 = b + 12 \end{cases}$$

Solving, $\boldsymbol{a = 32}$ **and** $\boldsymbol{b = 8}$

$\therefore$ **The total number of apples owned by them is 40 .** **[4]**

**8.** $\angle CAD = \angle CBD = 25°$

$\because AB = AD$

$\therefore \angle BAD = 180° - 2 \times 58° = 64°$

$\angle BDC = \angle BAC = 64° - 25° = \mathbf{39}°$

$\because BC = CE$

$\therefore \angle BEC = \dfrac{180° - 58°}{2} = 61°$

$\angle ABE = 61° - \angle BAE = \mathbf{22}°$ **[5]**

**9. (a)** Let x° be the required angle.

$\pi \times 12^2 \times \dfrac{x}{360°} = 30°$

$x = 75$

$\therefore$ The angle of the sector is **75°** . **[3]**

**(b)** The required perimeter

$= 2\pi(12)\left(\dfrac{75}{360}\right) + 2 \times 12$

$= (\mathbf{24 + 5\pi})$ **cm** **[3]**

## Section A(2) [35]

**10. (a)** Let $S = a + bn$

$$\begin{cases} a + 10b = 10600 \\ a + 6b = 9000 \end{cases}$$

Solving, we have $a = 6600$ and $b = 400$

$\therefore$ **The required income = 6600 + 20 × 400 = $14600** **[4]**

**(b)** Let $N$ be the number of handbags sold in a month such that her income is \$18000.

$$N = \frac{18000 - 6600}{400} = 28.5$$

$\because$ 28.5 is not an integer

$\therefore$**It is not possible that the income is \$18000.** **[2]**

**11. (a)** $f(2) = (2-2)^2(2+h)+k = -5$

$f(3) = (3-2)^2(3+h)+k = 0$

Solving, **we have $h = 2$ and $k = -5$** **[3]**

**(b)**

$$f(x) = 0$$
$$(x-2)^2(x+2) - 5 = 0$$
$$x^3 - 2x^2 - 4x + 3 = 0$$
$$(x-3)(x^2+x-1) = 0$$
$$x = 3 \text{ or } x = \frac{-1\pm\sqrt{5}}{2}$$

Note that $\frac{-1\pm\sqrt{5}}{2}$ are not integers.

$\therefore$Not all the roots of $f(x) = 0$ are integers.

$\therefore$**The claim is disagreed.** **[3]**

**12. (a)** Mean = **55 kg**

Median = **52kg**

Range = 79 – 40

= **39kg** **[3]**

**(b)** Let $a$ and $b$ be the required weights.

$$\frac{a+b+55\times 20}{22} = 56$$
$$a+b = 132$$

Note that $a = 80$ kg as the range is increased by 1kg.

$\therefore$**The weight of each of these students is 80 kg and 52 kg.** **[4]**

**13. (a)**

$AF = BF$ (given )

$AB = BC$ (properties of square )

$\angle ABE = \angle BCF = 90°$ (properties of square )

$\Delta ABE \cong \Delta BCF$ (RHS ) **[2]**

**(b)** By (a), $\angle AEB = \angle BFC$

$\therefore \angle AEB = 90° - \angle FBC$

$\therefore \angle BGE$

$= 180° - \angle GEB - \angle GBE$

$= 180 - (90° - \angle GBE) - \angle GBE$

$= 90°$

$\therefore \Delta BGE$ is a right-angled triangle. **[3]**

**(c)** Note that $CF = BE = 15$ cm

$\boldsymbol{BG = \sqrt{15^2 - 9^2} = 12}$ **cm** **[2]**

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**14. (a) (i)** $m_L$

$= -1 \div \dfrac{-1-23}{4-(-14)}$

$= \dfrac{3}{4}$

The coordinates of midpoint of $PQ$

$= \left(\dfrac{4-14}{2}, \dfrac{-1+23}{2}\right)$

$= (-5,11)$

The equation of $L$ is

$$y - 11 = \frac{3}{4}(x - (-5))$$

$\boldsymbol{3x - 4y + 59 = 0}$ **[3]**

**(ii)** Centre of $C = \left(h, \frac{3h+59}{4}\right)$

The required equation is

$$\boldsymbol{(x-h)^2 + \left(y - \frac{3h+59}{4}\right)^2 = (h-4)^2 + \left(\frac{3h+59}{4} - (-1)\right)^2}$$

$2x^2 - 4xh + 2h^2 + 2y^2 - 3hy - 59y + \dfrac{3481}{8} + \dfrac{9h^2}{8} + \dfrac{177}{4}h = \dfrac{25h^2}{8} + \dfrac{125}{4}h + \dfrac{4225}{8}$

$2x^2 + 2y^2 - 4hx - (3h+59)y + 13h - 93 = 0$ **[3]**

**(b)** Put $(26,43)$ into the equation of $C$, we have $h = 11$

Centre of $C = (11,23)$

The required diameter

$= 2\sqrt{(11-4)^2 + (23-(-1))^2}$

$= \mathbf{50}$ **[3]**

| **Section B** | **[35]** |
|---|---|

**15. (a)** Let $x$ marks be the required score.

$$\frac{x-66}{12} = -0.5$$

$$x = 60$$

**The score of David in the Mathematics examination is 60 marks.** **[2]**

**(b)** Note that $60 = 66 - 0.5 \times 12$ and $49 = 52 - 0.3 \times 10$

Since $-0.3 > -0.5$,

he performs better in the Science examination than in the mathematics examination.

$\therefore$ **The claim is agreed.** **[2]**

**16. (a)** The required probability

$$= \frac{C_2^5 C_2^9}{C_4^{14}}$$

$$= \mathbf{\frac{360}{1001}}$$ **[2]**

**(b)** The required probability

$$= 1 - \frac{C_1^5 C_3^9 + C_0^5 C_4^9}{C_4^{14}}$$

$$= \mathbf{\frac{5}{11}}$$ **[2]**

**17. (a)** $A(1) + A(2) + A(3) + \cdots + A(n)$

$$= \frac{n}{2}(-1 + (4n - 5))$$

$$= \mathbf{2n^2 - 3n}$$ **[2]**

**(b)** $\log(B(1)B(2)B(3)\cdots B(n))$

$$= \log\left(10^{A(1)+A(2)+A(3)+\cdots+A(n)}\right)$$

$$= \log\left(10^{2n^2-3n}\right)$$

$$= 2n^2 - 3n$$

$$2n^2 - 3n \geq 8000$$

$$2n^2 - 3n - 8000 \geq 0$$

$$-\frac{125}{2} \leq n \leq 64$$

$\therefore$ **The greatest value of $n$ is 64.** **[3]**

**18.** **(a)** $\Delta = (-4k)^2 - 4(2)(3k^2+5)$

$= -8(k^2+5)$

$< 0$

$\therefore$ $f(x) = 0$ does not have real roots

$\therefore$ **The graph of $y = f(x)$ does not cut the x-axis.** **[2]**

**(b)** $2x^2 - 4kx + 3k^2 + 5$

$= 2(x^2 - 2kx + k^2) + k^2 + 5$

$= 2(x-k)^2 + k^2 + 5$

$\therefore$ The coordinates of the vertex is $(\boldsymbol{k}, \boldsymbol{k^2+5})$ **[3]**

**(c)** Note that when $S$ and $T$ are nearest to each other, they coincide with the vertices of the graphs of $y = f(x)$ and $y = 2 - f(x)$ respectively.

$\therefore S(k, k^2+5)$ and $T(k, -k^2-3)$

The $y$-coordinate of the midpoint of $ST$

$= \frac{k^2+5-k^2-3}{2}$

$=1$

$\therefore$ The perpendicular bisector of $ST$ is $y = 1$

$\therefore$ **The claim is disagreed.** **[4]**

**19.** **(a)** **(i)** $AC^2 = 40^2 + 24^2 - 2(40)(24)\cos 80°$

$AC = 42.92546446$

$\boldsymbol{AC = 42.9}$ **cm** **[2]**

**(ii)** $\cos\angle ACB = \dfrac{24^2 + 42.92546446^2 - 40^2}{2(24)(42.92546446)}$

$\angle ACB = 66.59081487°$

$\boldsymbol{\angle ACB = 66.6°}$ **[1]**

**(iii)** Note that the area of $\Delta ABC$ and that of $\Delta ABD$ are fixed.

The area of $\Delta ACD = \frac{1}{2}AC^2 \sin\angle CAD$

When $\angle BCD$ increases from 105° to 145° ,

$\angle CAD$ decreases from 103° to 23.2° .

**The required area increases when $\angle BCD$ increases from 105° to 111.6° but decreases when it increases from 111.6° to 145°.** **[4]**

**(b)** Let $M$ be the projection of $B$ onto $CD$ and $N$ be that of $B$ onto the plane $ACD$

$CM = AC\cos\angle ACD = 17.86278929$ cm

$BM = \sqrt{BC^2 - CM^2} = 16.02874788$ cm

$AM = \sqrt{AC^2 - CM^2} = 39.03224638$ cm

By Heron's formula,

The area of $\Delta ABM = 309.5495007$ cm$^2$

$$\frac{BN \times AM}{2} = 309.5495007$$

$$\boldsymbol{BN = 15.86121883}\ \textbf{cm}$$

The required volume

$$= \frac{1}{3} \times \left(\frac{1}{2}(AC)^2 \sin\angle CAD\right) \times BN$$

$= 3690$ cm$^3$

**∴ The required volume is 3690 cm$^3$.** **[6]**