HKDSE Mathematics 2014 Core Paper 1–Suggested Solution

Section A(1)	**[35]**

1. $\dfrac{(xy^{-2})^3}{y^4}$

$= \dfrac{x^3y^{-6}}{y^4}$

$= \dfrac{x^3}{y^{4-(-6)}}$

$= \boldsymbol{\dfrac{x^3}{y^{10}}}$ **[3]**

2. (a) $a^2 - 2a - 3$

$= \boldsymbol{(a+1)(a-3)}$ **[1]**

(b) $ab^2 + b^2 + a^2 - 2a - 3$

$= ab^2 + b^2 + (a+1)(a-3)$

$= b^2(a+1) + (a+1)(a-3)$

$= \boldsymbol{(a+1)(b^2+a-3)}$ **[2]**

3. (a) **200** **[1]**

(b) **123** **[1]**

(c) **123.4** **[1]**

4. The median $=$ **1**

The mode $=$ **2**

The standard deviation $=$ **0.889** **[3]**

5. (a) $2(3m+n) = m+7$

$6m + 2n = m + 7$

$\boldsymbol{n = \dfrac{7-5m}{2}}$ **[2]**

(b) The decrease in the value of $n = 5$ **[2]**

6. (a) The selling price of the toy

$= 255(1-40\%)$

$= \$\mathbf{153}$ **[2]**

(b) Let $\$x$ be the cost of the toy.

$(1+2\%)x = 153$

$x = 150$

$\therefore$ **The cost of the toy is $\$\mathbf{150}$.** **[2]**

7. (a)

$$f(2) = -33$$

$$4(2)^23 - 5(2)^22 - 18(2) + c = -33$$

$$c = -9$$

$$f(-1)$$
$$= 4(-1)^3 - 5(-1)^2 - 18(-1) - 9$$
$$= 0$$

$\therefore$ **$x + 1$ is a factor of $f(x)$.** **[3]**

(b)

$$f(x) = 0$$

$$4x^3 - 5x^2 - 18x - 9 = 0$$

$$(x+1)(4x^2 - 9x - 9) = 0$$

$$(x+1)(x-3)(4x+3) = 0$$

$$x = -1, x = 3 \text{ or } x = -\frac{3}{4}$$

Note that -1 , 3 and $-\frac{3}{4}$ are rational numbers.

$\therefore$ **The claim is agreed.** **[2]**

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8. (a) P' $(\mathbf{5}, \mathbf{3})$

Q' $(\mathbf{-19}, \mathbf{-7})$ **[2]**

(b)

$$m_{PQ}$$
$$= \frac{5+7}{-3-2}$$
$$= -\frac{\mathbf{12}}{\mathbf{5}}$$

$$m_{P'Q'}$$
$$= \frac{3+7}{5+19}$$
$$= \frac{5}{12}$$

$\because$ $\boldsymbol{m_{PQ} \times m_{P'Q'} = -1}$ **.**

$\therefore$ $PQ \perp P'Q'$. **[3]**

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9. (a) In ΔABC and ΔBDC,

$\angle BAC = \angle DBC$	(given)
$\angle ACB = \angle BCD$	(common $\angle$)
$\angle ABC = \angle BDC$	($\angle$ sum of Δ)
$\Delta ABC \sim \Delta BDC$	(AAA) **[2]**

(b) By (a), $\frac{CD}{BC} = \frac{BC}{AC}$

$CD = 16$ cm

$BD^2 + CD^2$
$= 12^2 + 16^2$
$= 20^2$
$= BC^2$

$\therefore$ **ΔBCD is a right-angled triangle.** [3]

Section A(2) **[35]**

10. (a) The distance of car A from town X at 8:15 in the morning

$= \frac{45}{120}(80)$

$=$ **30 km** [2]

(b) Suppose that car A and car B first meet at the time t minutes after 7:30 in the morning.

$\frac{t}{120} = \frac{44}{80}$

$t = 66$

$\therefore$ **Car A and car B first meet at 8:36 in the morning.** [2]

(c) During the period 8:15 to 9:30 in the morning, car B travels 36 km while car A travels more than 36 km .

$\therefore$ The average speed of car A is greater than that of car B .

$\therefore$ **The claim is disagreed.** [2]

11. (a) The range $=$ **73 thousand dollars**

The inter-quartile range

$= 63 - 42$

$=$ **21 thousand dollars** [2]

(b) The mean of the prices of the remaining paintings in the art gallery

$= \frac{(33)(53) - 32 - 34 - 58 - 59}{33 - 4}$

$= \frac{1566}{29}$

$=$ **54 thousand dollars**

Note that 32 and 34 are less than 55 .

Also note that 58 and 59 are greater than 55 .

The median of the prices of the remaining paintings in the art gallery

= 55 thousand dollars [3]

12. **(a)** The radius of C

$= \sqrt{(6-0)^2 + (11-3)^2}$

$= 10$

$\therefore$ The equation of C is $\boldsymbol{x^2 + (y-3)^2 = 10^2}$ **[2]**

(b) **(i)** Let (x, y) be the coordinates of P.

$\sqrt{(x-0)^2 + (y-3)^2} = \sqrt{(x-6)62 + (y-11)^2}$

$3x + 4y - 37 = 0$

$\therefore$ **The equation of Γ is $3x + 4y - 37 = 0$** **[2]**

(ii) ***Γ* is the perpendicular bisector of the line segment *AG*.** **[1]**

(iii) The perimeter of the quadrilateral $AQGR$

$= 4(10)$

$= \mathbf{40}$ **[2]**

13. **(a)** **Let $f(x) = px^2 + q$**

$$\begin{cases} 4p + q = 59 \\ 49p + q = -121 \end{cases}$$

Solving, **we have $p = -4$ and $q = 75$**

$\therefore f(x) = 75 - 4x^2$

$\therefore \boldsymbol{f(6) = -69}$. **[4]**

(b) By (a), we have $a = -69$.

Since $f(x) = 75 - 4x^2$, we have $f(-6) = f(6)$.

So, we have $b = -69$.

AB

$= 6 - (-6)$

$= 12$

The area of ΔABC

$= \dfrac{(12)(69)}{2}$

$= \mathbf{414}$ **[4]**

14. **(a)** The slant height of the circular cone

$= \sqrt{72^2 + 96^2}$

$= 120$ cm

The area of the wet curved surface of the vessel

$= \pi(72)(120)\frac{(96-60+28)^2-(96-60)^2}{96^2}$

$= \pi(72)(120)\frac{64^2-36^2}{96^2}$

$= \mathbf{2625\pi\ cm^2}$ **[4]**

(b) The volume of the circular cone

$= \frac{1}{3}\pi(72)^2(96)$

$= 165888\pi\ \text{cm}^3$

The volume of water in the vessel

$= \mathbf{165888\pi\left(\frac{(64^3-36^3)}{96^3}\right)}$

$= 40404\pi\ \text{cm}^3$

$= 0.126932909\ \text{m}^3$

$> 0.1\ \text{m}^3$

$\therefore$ **The claim is agreed** **[4]**

Section B [35]

15. $\log_8 y - 0 = -\frac{1}{3}(\log_4 x - 3)$

$\log_8 y = -\frac{1}{3}\log_4 x + 1$

$\log_8 y = \log_4 x^{-\frac{1}{3}} + \log_4 4$

$\log_8 y = \log_4 4x^{-\frac{1}{3}}$

$\frac{\log_2 y}{\log_2 8} = \frac{\log_2 4x^{-\frac{1}{3}}}{\log_2 4}$

$\log_2 y = \frac{3}{2}\log_2 4x^{-\frac{1}{3}}$

$\log_2 y = \log_2 8x^{-\frac{1}{2}}$

$\mathbf{y = 8x^{-\frac{1}{2}}}$ **[3]**

16. Note that the numbers of dots in the patterns form an arithmetic sequence.

The total number of dots in the first m patterns

$= 3 + 5 + 7 + \cdots + (2m + 1)$

$= \mathbf{\frac{m}{2}(3 + (2m + 1))}$

$= m^2 + 2m$

$m^2 + 2m > 6888$

$m^2 + 2m - 6888 > 0$

$(m - 82)(m + 84) > 0$

$m < -84$ or $m > 82$

$\therefore$ **The least value of m is 83.** **[4]**

17. (a) By sine formula, we have

$$\frac{\sin \angle AVB}{AB} = \frac{\sin \angle VAB}{VB}$$

$\angle AVB = 34.32008291°$

$\angle VBA = 180° - 110° - 34.32008291°$

$\angle VBA = 35.67991709°$

$\boldsymbol{\angle VBA = 35.7°}$ **[2]**

(b) By cosine formula, we have

$MP^2 = BP^2 + BM^2 - 2(BP)(BM) \cos \angle VBA$

$MP^2 = 9^2 + 15^2 - 2(9)(15) \cos 35.67991709°$

$MP = 9.310329519$ cm

$$MN = \frac{BC}{2}$$

$MN = 5$ cm

Note that $MP = NQ$.

Let h cm be the height of the trapezium $PQNM$.

$$h = \sqrt{MP^2 - \left(\frac{PQ - MN}{2}\right)^2}$$

$$h = \sqrt{9.310329519^2 - \left(\frac{10 - 5}{2}\right)^2}$$

$h = 8.968402074$

The area of the trapezium $PQNM$

$$= \frac{h}{2}(MN + PQ)$$

$$= \frac{(8.968402074)(5 + 10)}{2}$$

$= 67.26301555 \text{ cm}^2$

$< 70 \text{ cm}^2$

$\therefore$ **The claim is agreed.** **[5]**

18. **(a)** m_{L_2}

$= \dfrac{90-0}{45-180}$

$= -\dfrac{2}{3}$

The equation of L_2 is:

$$y-90=-\frac{2}{3}(x-45)$$

$$\mathbf{2x+3y-360=0}$$

$\therefore$ The system of inequalities is $\begin{cases} \mathbf{6x+7y\le 900} \\ \mathbf{2x+3y\le 360} \\ \mathbf{x\ge 0} \\ \mathbf{y\ge 0} \end{cases}$ **[4]**

(b) Let x and y be the numbers of wardrobes X and Y produced that month respectively.

Now, the constraints are

$\begin{cases} \mathbf{6x+7y\le 900} \\ \mathbf{2x+3y\le 360} \end{cases}$, where x and y are non-negative integers.

Denote the total profit on the production of wardrobes by $\$P$.

$$\mathbf{P=440x+665y}$$

Note that the vertices of the shaded region in Figure 7 are the points $(0,0)$, $(0,120)$, $(45,90)$ and $(150,0)$.

For (0,0), $P=(440)(0)+(665)(0)=0$.

For $(0,120)$, $P=(440)(0)+(665)(120)=79800$.

For (45,90), $P=(440)(45)+(665)(90)=79650$.

For $(150,0)$, $P=(440)(150)+(665)(0)=66000$.

$\therefore$ The greatest possible total profit is $\$79800$.

$\therefore$ **The claim is disagreed.** **[4]**

19. **(a)** The required probability

$= \dfrac{1}{6}+\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{1}{6}\right)+\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{1}{6}\right)+\cdots$

$= \dfrac{\frac{1}{6}}{1-\frac{25}{36}}$

$= \mathbf{\dfrac{6}{11}}$ **[3]**

(b) **(i)** Suppose that the player of the second round adopts Option l.

The probability of getting 10 tokens

$= (1)\left(\frac{1}{8}\right)$

$= \frac{1}{8}$

The probability of getting 5 tokens

$= \frac{7(P_2^2)}{8^2}$

$= \frac{\mathbf{7}}{\mathbf{32}}$

The expected number of tokens got

$= (10)\left(\frac{1}{8}\right) + (5)\left(\frac{7}{32}\right)$

$= \frac{\mathbf{75}}{\mathbf{32}}$ **[4]**

(ii) Suppose that the player of the second round adopts Option 2.

The probability of getting 50 tokens

$= (1)\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)$

$= \frac{1}{64}$

The probability of getting 10 tokens

$= \frac{(6)(P_3^3)}{8^3}$

$= \frac{9}{128}$

The probability of getting 5 tokens

$= (2)\left(\frac{1}{8}\right)^2\left(\frac{1}{8}\right) + (6)\left(\frac{1}{8}\right)^2\left(\frac{2}{8}\right) + \left(\frac{7}{32}\right)\left(\frac{2}{8}\right)$

$= \frac{21}{256}$

The expected number of tokens got

$= (50)\left(\frac{1}{64}\right) + (10)\left(\frac{9}{128}\right) + (5)\left(\frac{21}{256}\right)$

$= \frac{485}{256}$

Note that $\frac{75}{32} > \frac{485}{256}$

$\therefore$ **The player of the second round should adopt Option l.** **[3]**

(iii) The probability of Ada getting no tokens

$= 1 - \left(\frac{6}{11}\right)\left(\frac{1}{8} + \frac{7}{32}\right)$

$= \frac{13}{16}$

< 0.9

$\therefore$ **The claim is incorrect**. **[3]**