# HKDSE Mathematics 2013 Core Paper 1–Suggested Solution

| **Section A(1)** | **[35]** |
|---|---|
| **1.** $\dfrac{x^{20}y^{13}}{(x^5y)^6}$ <br> $=\dfrac{x^{20}y^{13}}{x^{30}y^6}$ <br> $=\dfrac{y^{13-6}}{x^{30-20}}$ <br> $=\boldsymbol{\dfrac{y^7}{x^{10}}}$ | **[3]** |
| **2.** $\dfrac{3}{h}-\dfrac{1}{k}=2$ <br> $\dfrac{1}{k}=\dfrac{3}{h}-2$ <br> $\dfrac{1}{k}=\dfrac{3-2h}{h}$ <br> $\boldsymbol{k=\dfrac{h}{3-2h}}$ | **[3]** |
| **3.** **(a)** $4m^2-25n^2$ <br> $=\boldsymbol{(2m-5n)(2m+5n)}$ | **[1]** |
| **(b)** $4m^2-25n^2+6m-15n$ <br> $=(2m-5n)(2m+5n)+3(2m-5n)$ <br> $=\boldsymbol{(2m-5n)(2m+5n+3)}$ | **[2]** |
| **4.** Let \$$x$ and \$$y$ be the prices of a pear and an orange respectively. <br> $\begin{cases}\boldsymbol{7x+3y=47}\\\boldsymbol{5x+6y=49}\end{cases}$ <br> Solving, we have $x=5$ and $y=4$ <br> $\therefore$ **The required price is \$5** | **[4]** |
| **5.** **(a)** $\dfrac{19-7x}{3}>23-5x$ <br> $19-7x>69-15x$ <br> $-7x+15x>69-19$ <br> $\boldsymbol{x>\dfrac{25}{4}}$ | **[2]** |

| | | | |
|---|---|---|---|
| | **(b)** | $18 - 2x \geq 0$<br>$\boldsymbol{x \leq 9}$<br>By (a), we have $\frac{25}{4} < x \leq 9$<br>$\therefore$ **The required integers are 7, 8 and 9.** | **[2]** |
| **6.** | **(a)** | $L$ is the angle bisector of $\angle AOB$ | **[1]** |
| | **(b)** | Let $E(r, \theta)$ be the intersection point of $L$ and $AB$.<br>$\theta = \mathbf{70°}$<br>$r = OA \cos \angle EOA$<br>$= (26) \cos 60°$<br>$= \mathbf{13}$<br>$\therefore$ The required polar coordinates are $(13, 70°)$ | **[3]** |
| **7.** | **(a)** | $\angle BAC = \angle CDB$ (given )<br>$\angle DBC = \angle ACB$ (base ∠s, isos. Δ )<br>$BC = CB$ (common side )<br>$\Delta ABC \cong \Delta DCB$ (AAS ) | **[2]** |
| | **(b)** | **(i)** **3 pairs** | **[1]** |
| | | **(ii)** **4 pairs** | **[1]** |
| **8.** | **(a)** | The least possible weight of a regular pack of sea salt<br>$= 100 - 0.5$<br>$= \mathbf{99.5\ g}$ | **[2]** |
| | **(b)** | The least possible total weight of 32 regular packs of sea salt<br>$= (99.5)(32)$<br>$= \mathbf{3184\ g}$<br>$= 3.184$ kg<br>$= 3.2$ kg (correct to the nearest 0.1kg)<br>$> 3.1$ kg<br>$\therefore$ **The claim is not possible**. | **[3]** |
| **9.** | **(a)** | The mean $= \mathbf{3.5}$<br>The inter-quartile range $= 4 - 2 = \mathbf{2}$<br>The standard derivation $= \mathbf{1.5}$ | **[4]** |

**(b)** The new standard deviation $= 1.451456116$

The decrease in the standard deviation $= \mathbf{0.0485}$ **[1]**

## Section A(2) [35]

**10. (a)** The median$=$ $\mathbf{31}$

The mode$=$ $\mathbf{23}$ **[2]**

**(b) (i)** Note that $0 \le a < 5$ and $7 \le b \le 9$ .

Also note that the range of the distribution is 47.

$\therefore \begin{cases} \boldsymbol{a = 0} \\ \boldsymbol{b = 7} \end{cases}, \begin{cases} \boldsymbol{a = 1} \\ \boldsymbol{b = 8} \end{cases}$ **or** $\begin{cases} \boldsymbol{a = 2} \\ \boldsymbol{b = 9} \end{cases}$ **[2]**

**(ii)** The required probability

$= \dfrac{3+3+3+3+2+9+9}{260}$

$= \dfrac{32}{260}$

$= \dfrac{\mathbf{8}}{\mathbf{65}}$ **[2]**

**11. (a)** **Let** $\boldsymbol{W = hl + kl^2}$

$\therefore \begin{cases} h(1) + k(1^2) = 181 \\ h(2) + k(2^2) = 402 \end{cases}$

Solving, **we have** $\boldsymbol{h = 161}$ **and** $\boldsymbol{k = 20}$

The required weight

$= 161(1.2) + 20(1.2^2)$

$= \mathbf{222}$ **grams** **[4]**

**(b)** $20l^2 + 161l = 594$

$20l^2 + 161l - 594 = 0$

$\boldsymbol{l = \frac{11}{4}}$ **or** $\boldsymbol{l = -\frac{54}{5}}$ **(rejected)**

$\therefore$ The perimeter of the tray is $\frac{11}{4}$ m. **[2]**

**12. (a)** By comparing the coefficients of $x^3$ and the constant terms,

**we have $a = 3$ and $c = 4$**

Note that the coefficient of $x^2$ in the expansion of $(x-2)(3x^2+bx+4)$ is $b-6$.

By comparing the coefficients of $x^2$, we have $b-6=-7$.

$\therefore$ $b = -1$ . **[4]**

**(b)** $\Delta = (-1)^2 - 4(3)(4)$

$= -47$

$< 0$

$\therefore$ $3x^2 - x + 4 = 0$ has no real roots.

$\therefore$**The claim is disagreed.** **[3]**

**13. (a) (i)**

$$\frac{\pi r^2}{\pi R^2} = \frac{1}{9}$$

$$\frac{r}{R} = \frac{1}{3}$$

$r : R = 1 : 3$ **[2]**

**(ii)** Let $h$ cm be the height of a larger circular cylinder.

$$2\pi R^2 h = 27\left(\pi r^2 (10)\right)$$

$$h = \frac{270}{2}\left(\frac{r}{R}\right)^2$$

$$h = \frac{270}{2}\left(\frac{1}{3}\right)^2$$

$h = 15$

$\therefore$ The height of a larger circular cylinder is 15 cm . **[3]**

**(b)**

$$\frac{r}{R} = \frac{1}{3}$$

$$\frac{h_{\text{smaller cylinder}}}{h_{\text{larger cylinder}}} = \frac{10}{15} = \frac{2}{3}$$

$$\because \frac{r}{R} \neq \frac{h_{\text{smaller cylinder}}}{h_{\text{larger cylinder}}}$$

$\therefore$ The two circular cylinders are not similar

$\therefore$ **The claim is disagreed.** **[2]**

**14.** **(a)** The coordinates of R $= (\mathbf{6}, \mathbf{17})$ **[1]**

**(b)** **(i)** Let $(h, k)$ be the coordinates of $P$ .

Since $P$ lies on $L$ , we have $4h + 3k + 50 = 0$ .

$\because$ $RP \perp L$

$\therefore$ $m_{RP} \times m_L = -1$

$\left(\frac{k-17}{h-6}\right)\left(-\frac{4}{3}\right) = -1$

$3h - 4k + 50 = 0$ .

Solving, we have $h = -14$ and $k = 2$ .

$\therefore P\ (-14, 2)$

$PR = \sqrt{(-14-6)^2 + (2-17)^2}$

$= \mathbf{25}$ **[4]**

**(ii)** **(1)** $P$, $Q$ and $R$ are collinear. **[1]**

**(2)** Note that the radius of the $C$ is 10 .

QR= 10

$PQ = 25 - 10 = 15$

The required ratio

$= PQ : QR$

$= 15 : 10$

$= \mathbf{3 : 2}$ **[3]**

## Section B **[35]**

**15.** **(a)** Note that the highest score of the distribution is 90 marks.

Let $\mu$ marks and $\sigma$ marks be the mean and the standard deviation of the distribution respectively.

$$\begin{cases} 90 - \mu = 3\sigma \\ 65 - \mu = 0.5\sigma \end{cases}$$

Solving, we have $\boldsymbol{\mu = 60}$.

$\therefore$ The mean of the distribution is 60 marks. **[2]**

**(b)** Note that if the test score of a student is lower than the mean, then the standard score of the student is negative.

Also note that the median is 55 marks and the mean is 60 marks.

∴ The median < the mean.

∴ The test scores of at least half of the students < the mean.

**∴ The claim is agreed.** **[2]**

**16.** **(a)** The required probability

$$= \frac{C_4^5 C_2^{11} + C_5^5 C_1^{11}}{C_6^{16}}$$

$$= \frac{286}{8008}$$

$$= \mathbf{\frac{1}{28}}$$ **[2]**

**(b)** The required probability

$$= 1 - \frac{1}{28}$$

$$= \mathbf{\frac{27}{28}}$$ **[2]**

**17.** **(a)** $f(x)$

$= 36x - x^2$

$= -(x^2 - 36x + 18^2) + 18^2$

$= -(x - 18)^2 + 324$

**∴ The coordinates of the vertex are (18, 324)** **[2]**

**(b)** **(i)** $A$

$= x\left(\frac{108 - 3x}{2}\right)$

$= \mathbf{54x - \frac{3}{2}x^2}$ **[2]**

**(ii)** Note that $A = \frac{3}{2}f(x)$ , where $f(x) = 36x - x^2$ and $0 < x < 36$ .

By (a), the greatest value of $A$ is 486 .

**∴ The claim is disagreed.** **[2]**

**18.** **(a)** **(i)** Note that $\angle ABC = 90°$

$\tan \angle BCM = \frac{28}{21}$

$\angle BCM = 53.13010285°$

$\boldsymbol{\angle BCM = 53.1°}$ **[1]**

**(ii)** By sine formula,

$\frac{CM}{\sin \angle MBC} = \frac{BC}{\sin \angle BMC}$

$CM = 17.10154643 \text{ cm}$

$\boldsymbol{CM = 17.1 \text{ cm}}$ **[2]**

**(b)** **(i)** By cosine formula,

$AC^2 = AM^2 + CM^2 - 2(AM)(CM)\cos \angle AMC$

$AC^2 = (35 - 17.10154643)^2 + (17.10154643)^2 - 2(35 - 17.10154643)(17.10154643)\cos 107°$

$AC = 28.13898297$

$\boldsymbol{AC = 28.1 \text{ cm}}$ **[2]**

**(ii)** $CN$

$= CM \cos \angle BCM$

$= 17.10154643 \cos 53.13010235°$

$= 10.26092786 \text{ cm}$

By cosine formula,

$CN$

$= CM \cos \angle BCM$

$= 17.10154643 \cos 53.13010235°$

$= 10.26092786 \text{ cm}$

By cosine formula,

$AB^2 = BC^2 + AC^2 - 2(AC)(BC)\cos \angle ACE$

$\cos \angle ACB = \frac{21^2 + (28.13898297)^2 - 28^2}{2(28.13\,898297)(21)}$

$\angle ACB = 67.6818202°$

By cosine formula,

$AN^2 = CN^2 + AC^2 - 2(CN)(AC)\cos\angle ACB$

$AN^2 = (10.26092786)^2 + (28.13898297)^2 - 2(10.260927867)(28.13898297)\cos 67.6818202°$

$AN = 26.03453787$

By sine formula,

$$\frac{AC}{\sin\angle ANC} = \frac{AN}{\sin\angle ACN}$$

$$\frac{28.13898297}{\sin\angle ANC} = \frac{26.03453787}{\sin 67.6818202°}$$

$\angle ANC = 89.06498097°$ or $\angle ANC = 90.93501903°$

$\therefore$ $\angle ANC \neq 90°$

$\therefore$ $\angle ANM$ is not the angle between face $BCM$ and the horizontal ground.

$\therefore$ **The claim is disagreed.** **[3]**

**19. (a) (i)** The required area

$= \mathbf{9 \times 10^6(1 + r\%) - 3 \times 10^5}$

$= (870 + 9r) \times 10^4$ m$^2$ **[1]**

**(ii)** $(9 \times 10^6(1 + r\%) - 3 \times 10^5)(1 + r\%) - 3 \times 10^5 = 1.026 \times 10^7$

$150(1 + r\%)^2 - 5(1 + r\%) - 176 = 0$

$1 + r\% = \frac{11}{10}$ or $1 + r\% = -\frac{16}{15}$ (rejected )

$\therefore$ $\mathbf{r = 10}$ . **[3]**

**(b) (i)** The required area

$= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^5(1.1)^{n-2} - 3 \times 10^5(1.1)^{n-3} - 3 \times 10^5(1.1)^{n-4} - \cdots - 3 \times 10^5$

$= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^5\left(\frac{(1.1)^{n-1} - 1}{1.1 - 1}\right)$

$= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^6((1.1)^n - 1)$

$= \mathbf{(6(1.1)^{n-1} + 3) \times 10^6}$ **m**$^2$ **[3]**

**(ii)** $(6(1.1)^{n-1}+3)\times 10^6 > 4\times 10^7$

$(1.1)^{n-1} > \frac{37}{6}$ .

$\log(1.1)^{n-1} > \log\left(\frac{37}{6}\right)$ .

$(n-1)\log 1.1 > \log\left(\frac{37}{6}\right)$ .

$n > 20.08671715$ .

$\therefore$ **The total floor area of all public housing flats will first exceed $4\times 10^7$ m² at the end of the 21st year.** **[2]**

**(c)** Note that $a(1.21)^1 + b = 1\times 10^7$ and $a(1.21)^2 + b = 1.063\times 10^7$

Solving, we have $a = \frac{300}{121}\times 10^6$ and $b = 7\times 10^6$ .

$(6(1.1)^{n-1}+3)\times 10^6 > \left(\frac{300}{121}(1.21)^n + 7\right)\times 10^6 \ldots\ldots(*)$ .

$-\frac{300}{121}(1.21)^n - 7 + 6(1.1)^{n-1} + 3 > 0$

$75((1.1)^n)^2 - 165(1.1)^n + 121 < 0$

$\Delta$

$= (-165)^2 - 4(75)(121)$

$= -9075$

$< 0$

Since $75 > 0$, we have $75((1.1)^n)^2 - 165(1.1)^n + 121 > 0$ for all $n$.

$\therefore$ There is no solution for $(*)$ .

$\therefore$ **The claim is incorrect.** **[4]**