HKDSE Mathematics 2012 Core Paper 1–Suggested Solution

Section A(1)	**[35]**
1. $\dfrac{m^{-12}n^8}{n^3}$ $= \dfrac{n^{8-3}}{m^{12}}$ $= \dfrac{\boldsymbol{n^5}}{\boldsymbol{m^{12}}}$	**[3]**
2. $\dfrac{3a+b}{8} = b-1$ $3a+b = 8(b-1)$ $3a+b = 8b-8$ $3a = 7b-8$ $\boldsymbol{a = \dfrac{7b-8}{3}}$	**[3]**
3. (a) $x^2-6xy+9y^2$ $= \boldsymbol{(x-3y)^2}$	**[1]**
(b) $x^2-6xy+9y^2+7x-21y$ $=(x-3y)^2+7x-21y$ $=(x-3y)^2+7(x-3y)$ $= \boldsymbol{(x-3y)(x-3y+7)}$	**[2]**
4. (a) The daily wage of Ada $= 480(1+20\%)$ $= \$\boldsymbol{576}$	**[2]**
(b) Let $\$x$ be the daily wage of Christine. $x(1-20\%) = 480$ $x = \dfrac{480}{1-20\%}$ $x = 600$ $\therefore$ **Christine has the highest daily wage.**	**[2]**
5. **Let $\boldsymbol{x}$ and $\boldsymbol{(x+24)}$ be the number of male and female guards respectively.** $\boldsymbol{x+(x+24) = 132}$ $2x = 108$ $x = 54$ $\therefore$ **The number of male guards in the exhibition centre is 54.**	**[4]**

6. (a)

$\frac{4x+6}{7} > 2(x-3)$

$4x+6 > 14(x-3)$

$10x < 48$

$\boldsymbol{x < \frac{24}{5}}$

Also,

$2x - 10 \leq 0$

$\boldsymbol{x \leq 5}$

$\therefore$ The required solution is $x < \frac{24}{5}$ **[3]**

(b) **4** **[1]**

7. (a)

a

$= 18.1 - 6.8$

$= \boldsymbol{11.3}$

b

$= 12.1 + 3.2$

$= \boldsymbol{15.3}$ **[2]**

(b) Note that the longest time taken by the students to finish a 100m race after the training is 15.2 s which is less than the upper quartile of the distribution of the times taken before the training.

$\therefore$ The claim is agreed. **[2]**

8. (a)

$\boldsymbol{\Delta AED \sim \Delta BEC}$

$\frac{AE}{BE} = \frac{DE}{CE}$

$\frac{AE}{8} = \frac{15}{20}$

$\boldsymbol{AE = 6}$ **cm** **[3]**

(b)

$AE^2 + BE^2$

$= 6^2 + 8^2$

$= 10^2$ **[2]**

$= AB^2$

$\boldsymbol{\therefore AC \perp BD}$

9. **(a)** Let x cm be the length of AD .

$$\frac{(6+x)(12)}{2}(10)=1020$$

$$x=11$$

$\therefore$ The length of AD is 11cm . **[2]**

(b) CD

$= \sqrt{12^2+(11-6)^2}$

$= 13$ cm

The total surface area of the prism $ABCDEFGH$

$= (12+11+13+6)(10)+\frac{(6+11)(12)}{2}(2)$

$= \mathbf{624\ cm^2}$ **[3]**

Section A(2) [35]

10. **(a)** The mean

= 18

The median

= 16 **[2]**

(b) **(i)** The mean

= 18 **[2]**

(ii) Let a and b be the numbers of hours recorded in the two other questionnaires.

$$\frac{a+b+19+20}{4}=18$$

$$a+b=33$$

If the two medians are the same, then we have $a\le 16$ and $b\le 16$.

$a+b\le 32\ne 33$

$\therefore$ **It is not possible that the two medians are the same**. **[3]**

11. **(a)** **Let $C=r+sA$, where r and s are· non-zero constants.**

$$\therefore \begin{cases} r+2s=62 \\ r+6s=74 \end{cases}$$

Solving, we have $r=56$ **and** $s=3$.

The required cost

$= 56+3(13)$

$= \$\mathbf{95}$ **[4]**

(b) Since the volume of the larger can is 8 times that of the can described in (a), the surface area of the larger can is 4 times that of the can described in (a).

The surface area of the larger can

$= (13)(4)$

$= 52 \text{ m}^2$

The required cost

$= 56 + 3(52)$

$= \$\mathbf{212}$ **[2]**

12. (a) The required volume

$= \frac{1}{3}\pi(48)^2(96)$

$= \mathbf{73728\pi \text{ cm}^3}$ **[2]**

(b) (i) The required volume

$= \frac{2}{3}\pi(60)^3$

$= \mathbf{144000\pi \text{ cm}^3}$ **[2]**

(ii) Let h cm be the height of the frustum under the surface of the milk and r cm be the base radius of the circular cone above the surface of the milk.

h

$= \sqrt{60^2 - 48^2}$

$= 36$

$$\frac{r}{48} = \frac{96-36}{96}$$

$r = 30$

The volume of the milk remaining in the vessel

$= 144000\pi - \left(73728\pi - \frac{1}{3}\pi(30)^2(96-36)\right)$

$= 88272\pi \text{ cm}^3$

$= 0.2773146667 \text{ m}^3$

$< 0.3 \text{ m}^3$

$\therefore$ **The claim is disagreed.** **[3]**

13. (a) $k(2)^3 - 21(2)^2 + 24(2) - 4 = 0$

$8k = 40$

$\mathbf{k = 5}$ **[2]**

(b) (i) The area of the rectangle $OPQR$

$= \boldsymbol{m(15m^2 - 63m + 72)}$

$= 15m^3 - 63m^2 + 72m$ **[1]**

(ii) Note that the area of the rectangle $OPQR$ is 12.

$15m^3 - 63m^2 + 72m = 12$

$5m^3 - 21m^2 + 24m - 4 = 0$

$\boldsymbol{(m-2)(5m^2 - 11m + 2) = 0}$

$(m-2)^2(5m-1) = 0$

$m = 2$ or $m = \frac{1}{5}$

∴ There are only two different positions of Q such that the area of the rectangle $OPQR$ is 12.

∴ There are no three different positions of Q such that the area of the rectangle $OPQR$ is 12 . **[4]**

14. (a) (i) $\boldsymbol{\Gamma // L}$ **.** **[1]**

(ii) Note that the **y-intercept of Γ is -2 .**

m_L

$= \frac{-1-0}{0-3}$

$= \boldsymbol{\frac{1}{3}}$

The equation of Γ is

$y + 2 = \frac{1}{3}(x - 0)$

$\boldsymbol{x - 3y - 6 = 0}$ **[4]**

(b) (i) **Note that $\boldsymbol{Q(6,0)}$.**

Since $\boldsymbol{6 - 3(0) - 6 = 0}$, $\boldsymbol{\Gamma}$ passes through $\boldsymbol{Q}$. **[2]**

(ii) Note that both QH and QK are radii of the circle.

Also note that both the heights of ΔAQH and ΔBQK are the distance between L and Γ .

∴ area of ΔAQH = area of ΔBQK .

∴ The required ratio is 1:1 . **[2]**

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Section B	**[35]**

15. **(a)** The standard deviation

$= 10(1 + 20\%)$

$= \mathbf{12\ marks}$ **[1]**

(b) Let x be the test score and m be the mean of the test scores before the score adjustment.

The standard score before the score adjustment

$$= \frac{x - m}{10}$$

The standard score after the score adjustment

$$= \frac{(x(1 + 20\%) + 5) - (m(1 + 20\%) + 5)}{12}$$

$$= \frac{1.2(x - m)}{12}$$

$$= \frac{x - m}{10}$$

$\therefore$ **There is no change in the standard score of each student due to the score adjustment.** **[2]**

16. **(a)** The required probability

$$= \frac{C_4^8 (C_1^2)^4}{C_4^{16}}$$

$$= \mathbf{\frac{8}{13}}$$ **[2]**

(b) The required probability

$$= 1 - \frac{8}{13}$$

$$= \mathbf{\frac{5}{13}}$$ **[2]**

17. **(a)** Note that the radius of C is 10.

$\therefore$ **The equation of C is** $(\boldsymbol{x} - \mathbf{6})^2 + (\boldsymbol{y} - \mathbf{10})^2 = \mathbf{10^2}$ **[2]**

(b) The equation of L is $y = -x + k$.

Putting $y = -x + k$ in $x^2 + y^2 - 12x - 20y + 36 = 0$, we have

$x^2 + (-x + k)^2 - 12x - 20(-x + k) + 36 = 0$

$2x^2 + (8 - 2k)x + (k^2 - 20k + 36) = 0.$

The x-coordinate of the mid-point of AB

$$= \frac{\frac{-(8-2k)}{2}}{2}$$

$$= \frac{\boldsymbol{k-4}}{\boldsymbol{2}}$$

The y-coordinate of the mid-point of AB

$$= -\frac{k-4}{2} + k$$

$$= \frac{\boldsymbol{k+4}}{\boldsymbol{2}}$$

$\therefore$ The required coordinates are $\left(\frac{k-4}{2}, \frac{k+4}{2}\right)$.

[5]

18. **(a)** By sine formula, we have

$$\frac{AP}{\sin \angle PBA} = \frac{AB}{\sin \angle APB}$$

$$\frac{AP}{\sin 60°} = \frac{20}{\sin(180° - 60° - 72°)}$$

$$AP = 23.30704256 \text{ cm}$$

$$\boldsymbol{AP = 23.3 \text{ cm}}$$

$\therefore$ The length of AP is 23.3 cm .

[2]

(b) **(i)** Let S be the foot of the perpendicular from P to AD.

PS

$= AP \sin \angle PAD$

$= 23.30704256 \sin 72°$

$= 22.1663147$ cm

AS

$= AP \cos \angle PAD$

$= 23.30704256 \cos 72°$

$= 7.202272239$ cm

By sine formula, we have

$$\frac{PB}{\sin \angle PAB} = \frac{AB}{\sin \angle APB}$$

$$\frac{PB}{\sin 72°} = \frac{20}{\sin(180° - 60° - 72°)}$$

$$PB = 25.59545552 \text{ cm}$$

Let T be the foot of the perpendicular from P to BC.

$PT^2 = PB^2 - AS^2$

$PT^2 = (25.59545552)^2 - (7.202272239)^2$

$PT = 24.56124219$ cm

Note that $\alpha = \angle PTS$

By cosine formula, we have

$$\cos\alpha = \frac{PT^2 + ST^2 - PS^2}{2(PT)(ST)}$$

$$\cos\alpha = \frac{(24\ 56124219)^2 + 20^2 - (22.1663147)^2}{2(24\ 56124219)(20)}$$

$\alpha = 58.59703733°$

$\boldsymbol{\alpha = 58.6°}$ **[4]**

(ii) Let X be the projection of P on the base $ABCD$.

Then, we have $\beta = \angle PBX$

Note that $PB > PT$.

$\sin\alpha$

$= \frac{PX}{PT}$

$> \frac{PX}{PB}$

$= \sin\angle PBX$

$= \sin\beta$

Since α and β are acute angles, α is greater than β. **[2]**

19. (a) (i) Note that $ab^2 = 254100$ and $ab^4 = 307461$

So, we have $b^2 = \frac{307461}{254100}$

Solving, we have $\boldsymbol{b = 1.1}$ **and** $\boldsymbol{a = 210000}$.

The required weight

$= (210000)(1.1^{(2)(4)})$

$= \mathbf{450153.6501}$ **tonnes** **[4]**

(ii) The total weight of the goods

$= ab^2 + ab^4 + \cdots + ab^{2n}$

$= \frac{ab^2(b^{2n} - 1)}{b^2 - 1}$

$= \frac{(210000)(1.1)^2((1.1)^{2n}-1)}{1.1^2-1}$

$= \mathbf{1210000((1.1)^{2n}-1)}$ **tonnes** **[2]**

(b) (i) Note that $A(4) = 450153.65 > 420\,000 = 2a$.

Also note that $(1.1)^{2m} > (1.1)^m$ for any positive integer m .

$A(m+4)$

$= (1.1)^{2m}A(4)$

$> (1.1)^{2m}(2a)$

$> (1.1)^m(2a)$

$= B(m)$

$\therefore$ **The claim is agreed.** **[2]**

(ii) Let n be the number of years elapsed since the start of the operation of X.

The total weight of the goods handled by Y

$= 2ab + 2ab^2 + \cdots + 2ab^{n-4}$

$= \left(\frac{2ab(b^{n-4}-1)}{b-1}\right)$ tonnes, where $n > 4$

$$\mathbf{1210000((1.1)^{2n}-1) + \frac{420000(1.1)((1.1)^{n-4}-1)}{1.1-1} > 20000000}$$

$$121(1.1^{2n}) + 462(1.1^{n-4}) - 2583 > 0$$

$$121(1.1^4)(1.1^n)^2 + 462(1.1^n) - 2583(1.1^4) > 0$$

$1.1^n > 3.496831134$ or $1.1^n < -6.104\,70069$ (rejected)

$n \log 1.1 > \log 3.496831134$

$n > 13.13455888$

Note that n is an integer.

$\therefore$ **The new facilities should be installed in the 14th year since the start of the operation of X.** **[5]**